Topic : Limit Problems based on logarithmic Series expansion

* $\log_e(1+x)$

PDF link : Description .

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Q.1: $\lim_{x \to 0} \frac{\log(1+x)}{x}$ $\quad \frac{\log(1+0)}{0} = \frac{\log 1}{0} = \frac{0}{0}$ (Indeterminate form / अनिर्धार्य रूप)

$$= \lim_{x \to 0} \left[\frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}{x} \right]$$

$$= \lim_{x \to 0} \frac{x\left[1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots\right]}{x}$$

$$= \lim_{x \to 0} 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots$$

$$= 1$$

Q.2: $\lim_{x\to 1} \frac{\log x}{1-x}$ $\quad \frac{\log 1}{1-1} = \frac{0}{0}$ (Indeterminate form
अनिर्धार्य रूप)

$\lim_{(x-1)\to 0} \frac{\log x}{1-x}$

let $x-1=h$ as $x\to 1$ then $h\to 0$

$x = 1+h$

$\lim_{h\to 0} \frac{\log(1+h)}{1-(1+h)} = \lim_{h\to 0} \frac{\log(1+h)}{1-1-h} = \lim_{h\to 0} \frac{\log(1+h)}{-h}$

$= \lim_{h\to 0} \frac{h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \cdots}{-h}$

$= \lim_{h\to 0} \frac{h\left(1 - \frac{h}{2} + \frac{h^2}{3} - \frac{h^3}{4} + \cdots\right)}{-h} = \lim_{h\to 0} -\left(1 - \frac{h}{2} + \frac{h^2}{3} - \frac{h^3}{4} + \cdots\right)$

$= -1$

Q.3: $\lim_{x \to 1} \frac{\log x}{x-1}$ $\quad \frac{\log 1}{1-1} = \frac{0}{0}$ (Indeterminate form / अनिर्धार्य रूप)

$$\lim_{(x-1)\to 0} \frac{\log x}{x-1}$$

let $x-1 = h$ when $x \to 1$ then $h \to 0$

$x = 1+h$

$$\lim_{h\to 0} \frac{\log(1+h)}{1+h-1} = \lim_{h\to 0} \frac{\log(1+h)}{h}$$

$$= \lim_{h\to 0} \frac{h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \cdots}{h} = \lim_{h\to 0} \frac{h\left(1-\frac{h}{2}+\frac{h^2}{3}-\frac{h^3}{4}+\cdots\right)}{h}$$

$$= \lim_{h\to 0} 1 - \frac{h}{2} + \frac{h^2}{3} - \frac{h^3}{4} + \cdots = 1$$

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + - - -$$

Replace $x \to -x$ in both side of above equation we have.

$$\log(1+(-x)) = (-x) - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \frac{(-x)^4}{4} + - - -$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}$$

Q.4 : $\lim_{x \to 0} \frac{\tan x}{x}$ $\quad \frac{\tan 0}{0} = \frac{0}{0}$ (Indeterminate form / अनिर्धार्य रूप)

$$= \lim_{x \to 0} \frac{1}{x} \frac{\sin x}{\cos x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$= \frac{1}{1}$$

$$\left[\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \cos x = 1 \right]$$

$$= 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

Q.5: $\lim_{x\to 0} \frac{x}{\tan x}$ $\quad \frac{0}{\tan 0} = \frac{0}{0}$ (Indeterminate form / अनिर्धार्य रूप)

$$= \lim_{x\to 0} \frac{x}{\frac{\sin x}{\cos x}}$$

$$= \lim_{x\to 0} \frac{x \cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{\cos x}{\frac{\sin x}{x}}$$

$$= \frac{1}{1}$$

$$= 1$$

$$\lim_{x\to 0} \frac{x}{\tan x} = 1$$

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