Topic :

① Limit's Problem based on Trigonometric simplification.

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Results Used :

1. $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1$$

2. $$\lim_{x \to 0} \sin x = 0$$

3. $$\lim_{x \to 0} \cos x = 1$$

4. $$\lim_{x \to 0} \tan x = 0$$

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Q.1: $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$ $\frac{\sin 0}{\sin 0}$ $\frac{0}{0}$

$$= \lim_{x \to 0} \frac{ax \frac{\sin ax}{ax}}{bx \frac{\sin bx}{bx}}$$

$$= \lim_{x \to 0} \frac{ax \left[\frac{\sin ax}{ax}\right]}{bx \left[\frac{\sin bx}{bx}\right]}$$

$$= \frac{a}{b} \quad \frac{1}{1} \qquad = \frac{a}{b}$$

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$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos x = 1 - 2\sin^2 \frac{x}{2}$$

$$2\sin^2 \frac{x}{2} = 1 - \cos x$$

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Q.2 : $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ $\quad \frac{1-1}{0^2} \quad \frac{0}{0}$

$$\lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{2^2 \, \frac{x^2}{2^2}}$$

$$\lim_{x \to 0} \frac{\sin ax}{ax} = 1$$

$$\lim_{x \to 0} \frac{2}{4} \frac{\left(\sin \frac{x}{2}\right)^2}{\left(\frac{x}{2}\right)^2}$$

$$\lim_{x \to 0} \frac{1}{2} \left[\frac{\sin x/2}{x/2} \right]^2$$

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

$$\frac{1}{2} (1)^2 = \frac{1}{2}$$

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Q.3: $\lim_{x\to 0} \frac{1-\cos 6x}{x}$ $\quad \frac{1-1}{0}$ $\quad \frac{0}{0}$

$\cos 6x = 1 - 2\sin^2 3x$

$2\sin^2 3x = 1 - \cos 6x$

$$\lim_{x\to 0} \frac{2\sin^2 3x}{x}$$

$$\lim_{x\to 0} 2\frac{(\sin 3x)^2}{3^2\, x \times x} \times x \times 3^2$$

$$\lim_{x\to 0} 2 \frac{(\sin 3x)^2}{(3x)^2}\, 9x$$

$$\lim_{x\to 0} 2\left(\frac{\sin 3x}{3x}\right)^2 9x$$

$$2 \times (1)^2\ 9 \times 0 = 0$$

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Q.4: $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$ $\qquad \frac{\tan 0 - \sin 0}{0^3} = \frac{0-0}{0} = \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \frac{\sin x}{1}}{x^3}$$

$$\lim_{x\to 0} \frac{\sin x - \cos x \sin x}{\cos x \; x^3}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \frac{(1-\cos x)}{x^2 \cos x}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \frac{2\sin^2 x/2}{2^2 \frac{x^2}{2^2} \cos x} = \lim_{x\to 0} \frac{\sin x}{x} \frac{2}{4} \frac{(\sin x/2)^2}{(x/2)^2} \frac{1}{\cos x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \frac{1}{2} \left(\frac{\sin x/2}{x/2}\right)^2 \frac{1}{\cos x} = 1 \times \frac{1}{2} \times (1)^2 \frac{1}{1} = \frac{1}{2}$$

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$$\sin 2x = 2\sin x \cos x$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$$

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Q.5: $\lim_{x\to\frac{\pi}{2}} (\sec x - \tan x)$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}} ;$$

$$= \lim_{x\to\pi/2} \frac{(\cos x/2 - \sin x/2)^2}{(\cos x/2 + \sin x/2)(\cos x/2 - \sin x/2)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x/2 - \sin x/2}{\cos x/2 + \sin x/2}$$

$$= \frac{\cos \pi/4 - \sin \pi/4}{\cos\frac{\pi}{4} + \sin \pi/4}$$

$$= \frac{1/\sqrt{2} - 1/\sqrt{2}}{\frac{1}{\sqrt{2}} + 1/\sqrt{2}}$$

$$= 0$$

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Q.6 : $\lim\limits_{x \to 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$ $\quad (1-1) \tan\frac{\pi}{2} = 0 \times \infty$

$= \lim\limits_{(x-1) \to 0} (1-x) \tan\frac{\pi x}{2}$

$x-1 = h$ $\quad$ then $h \to 0$

$x = 1+h$

$= \lim\limits_{h \to 0} (1-(1+h)) \tan\left(\frac{\pi}{2}(1+h)\right) = \lim\limits_{h \to 0} -h \tan\left(\frac{\pi}{2} + \frac{h\pi}{2}\right)$

$= \lim\limits_{h \to 0} -h \left(-\cot \frac{h\pi}{2}\right) = \lim\limits_{h \to 0} h \cot\left(\frac{h\pi}{2}\right)$

$= \lim\limits_{h \to 0} h \frac{\cos(h\pi/2)}{\sin(h\pi/2)} = \lim\limits_{h \to 0} \frac{h\pi}{2} \times \frac{2}{\pi} \frac{\cos(h\pi/2)}{\sin h\pi/2}$

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$$\lim_{h\to 0} \frac{h\pi}{2} \times \frac{2}{\pi} \frac{\cos(h\pi/2)}{\sin h\pi/2}$$

$$= \lim_{h\to 0} \frac{2}{\pi} \frac{\cos(h\pi/2)}{\frac{\sin(h\pi/2)}{(h\pi/2)}}$$

$$= \lim_{h\to 0} \frac{2}{\pi} \times \frac{1}{1} = \frac{2}{\pi}$$

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Q.7: $\lim_{x \to \frac{\pi}{2}} \dfrac{2x-\pi}{\cos x}$

$\dfrac{\cancel{2} \times \frac{\pi}{\cancel{2}} - \pi}{\cos \pi/2}$ $\quad \dfrac{0}{0}$

$= \lim_{(x-\frac{\pi}{2}) \to 0} \dfrac{2x-\pi}{\cos x}$

let $x - \frac{\pi}{2} = h$ then $h \to 0$

$x = \frac{\pi}{2} + h$

$= \lim_{h \to 0} \dfrac{2\left(\frac{\pi}{2}+h\right) - \pi}{\cos\left(\frac{\pi}{2}+h\right)}$

$= \lim_{h \to 0} \dfrac{\pi + 2h - \pi}{-\sin h}$

$\lim_{h \to 0} \dfrac{-2h}{\sin h}$

$= \lim_{h \to 0} -2 \dfrac{1}{\frac{\sin h}{h}}$

$= -\dfrac{2}{1}$

$= -2$

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Factorial (क्रमगुणित) :

Notation (संकेत) !

Examples (उदाहरण) :

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$n! = n(n-1)(n-2)(n-3) \cdots 1$

Other Notation of Factorial $\lfloor\ $ ex: $\underline{|n}$

$1! = 1$ $\quad$ $0! = 1$

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Series expansion $\sin x$ and $\cos x$:

(odd Function) (Even function)

$$\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - -$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - - -$$

$$1 = \frac{x^0}{0!}$$

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Q.8: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$= \lim_{x \to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ---}{x}$$

$$= \lim_{x \to 0} \frac{x\left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + ---\right]}{x}$$

$$= \lim_{x \to 0} 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + ---$$

$$= 1$$

$$\lim_{x \to 0} \frac{x}{\sin x} = 1$$

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$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \to 0} \frac{1}{\left(\frac{\sin x}{x}\right)}$$

$$\frac{1}{1}$$

$$1$$

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Q.9 : $\lim_{x \to 0} \sin x = 0$

$$= \lim_{x \to 0} x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - -$$

$$= 0 - \frac{0^3}{3!} + \frac{0^5}{5!} - \frac{0^7}{7!} + - - -$$

$$= 0$$

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Q.10: $\lim_{x \to 0} \cos x = 1$

$$= \lim_{x \to 0} 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$= 1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \cdots$$

$$= 1$$

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Q.11 : $\lim_{x \to 0} \tan x = 0$

$= \lim_{x \to 0} \frac{\sin x}{\cos x}$

$= \frac{0}{1}$

$= 0$

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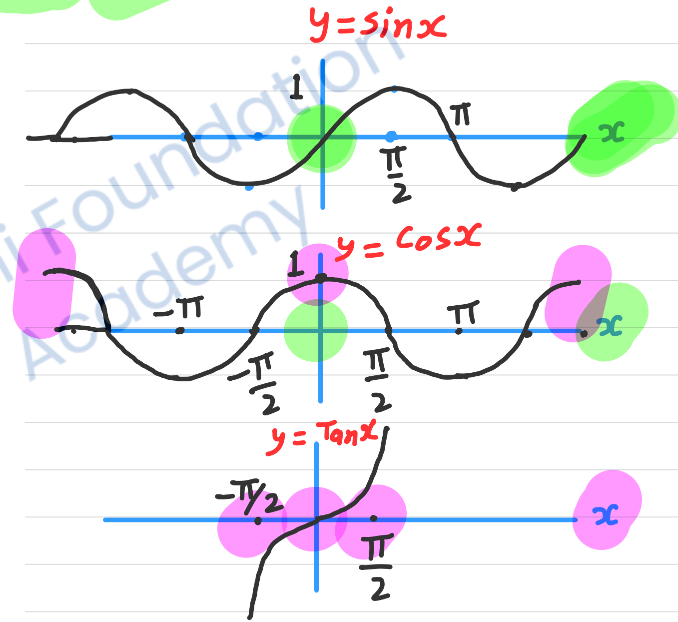
Summary:

1. $$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{x}{\sin x} = 1$$

2. $$\lim_{x \to 0} \sin x = 0$$

3. $$\lim_{x \to 0} \cos x = 1$$

4. $$\lim_{x \to 0} \tan x = 0$$



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