Topic : Limit's Problem based on Algebric Simplification.

(बीजगणितीय सरलीकरण पर आधारित सीमा के सवाल)

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Limit : lim Lt

$f(x) = x^2$

$f(2) = 2^2 = 4$

$\lim_{x \to 2} f(x) = 4$

$\lim_{x \to 2^-} f(x)$

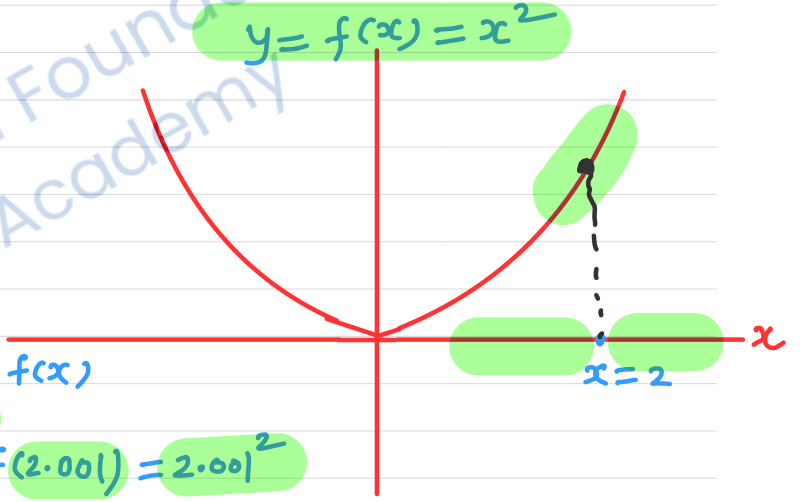
$\lim_{x \to 1.999} f(1.999) = 1.999^2$

$= 3.999$

$\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2.001} f(2.001) = 2.001^2$

$= 4.004$



Algebraic Simplification (बीजगणितीय सरलीकरण)

1. $a^2 - b^2 = (a+b)(a-b)$
2. $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
3. $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
4. Factorisation (गुणनखंड)
5. Rationalisation (परिमेयीकरण)

Type : 1 $\frac{0}{0}$ (Indeterminate form)

(अनिर्धार्य रूप)

- use Algebraic Simplification method

Q.1: $\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{0}{0}$

$= \lim_{x \to 2} \frac{x^2-2^2}{x-2}$

$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)}$ $\quad x \neq 2$

$= \lim_{x \to 2} x+2$

$= 2+2$

$= 4$

Q.2: $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{3^2 - 9}{3 - 3} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{x^2 - 3^2}{x - 3}$

$= \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)}$; $x \neq 3$

$= \lim_{x \to 3} x + 3$

$= 3 + 3$

$= 6$

Q.3: $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2} = \frac{2^2 - 3(2) + 2}{2 - 2} = \frac{4 - 6 + 2}{0} = \frac{0}{0}$

$= \lim_{x \to 2} \frac{x^2 - 2x - x + 2}{x - 2}$

$= \lim_{x \to 2} \frac{x(x-2) - 1(x-2)}{(x-2)}$

$= \lim_{x \to 2} \frac{(x-2)(x-1)}{(x-2)}$

$= \lim_{x \to 2} x - 1$

$= 2 - 1 = 1$

Q.4 : $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \frac{2^2 + 2 - 6}{2^2 - 4} = \frac{0}{0}$

$= \lim_{x \to 2} \frac{x^2 + 3x - 2x - 6}{x^2 - 2^2}$

$= \lim_{x \to 2} \frac{x(x+3) - 2(x+3)}{(x+2)(x-2)}$

$= \lim_{x \to 2} \frac{(x+3)(x-2)}{(x+2)(x-2)} \quad x \neq 2$

$= \lim_{x \to 2} \frac{x+3}{x+2} = \frac{2+3}{2+2} = \frac{5}{4}$

Q.5: $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x^2 - 1} = \frac{2(1)^2 - 3(1) + 1}{(1)^2 - 1} = \frac{0}{0}$

$= \frac{1}{2}$

Q. 6 : $\lim_{x \to 1} \frac{3x^2 - 4x + 1}{x^2 - 4x + 3} = \frac{3(1)^2 - 4(1) + 1}{1^2 - 4(1) + 3} = \frac{3 - 4 + 1}{1 - 4 + 3} = \frac{0}{0}$

$= \lim_{x \to 1} \frac{3x^2 - 3x - x + 1}{x^2 - 3x - x + 3}$

$= \lim_{x \to 1} \frac{3x(x-1) - 1(x-1)}{x(x-3) - 1(x-3)}$

$= \lim_{x \to 1} \frac{(x-1)(3x-1)}{(x-3)(x-1)} \; ; \; x \neq 1$

$= \lim_{x \to 1} \frac{3x-1}{x-3} = \frac{3(1)-1}{(1)-3} = \frac{2}{-2} = -1$

Q.7 : $\lim_{x \to 3} \frac{x^4 - 81}{x^3 - 27} = \frac{3^4 - 81}{3^3 - 27} = \frac{81 - 81}{27 - 27} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{(x^2)^2 - (9)^2}{x^3 - 3^3}$

$= \lim_{x \to 3} \frac{(x^2 + 9)(x^2 - 9)}{(x - 3)(x^2 + 3^2 + 3x)}$ $\quad x^2 - 3^2$

$= \lim_{x \to 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{(x - 3)(x^2 + 9 + 3x)}$; $x \neq 3$

$= \frac{(9 + 9)(6)}{9 + 9 + 9} = \frac{2 \cancel{18} \times 6^2}{\cancel{27}_3} = 4$

Q.8 : $\lim_{x \to a} \frac{x^3 - a^3}{x - a} = \frac{a^3 - a^3}{a - a} = \frac{0}{0}$

$= 3a^2$

Limit of Irrational Function :

(अपरिमेय फलन की सीमा)

→ आवश्यकता अनुसार हर या अंश का परिमेयीकरण करें

(Denominator) (Numerator) (Rationalisation)

Q.9: $\lim_{x \to 2} \frac{\sqrt{3-x} - 1}{2-x} = \frac{\sqrt{3-2} - 1}{2-2} = \frac{0}{0}$

$(a - b)(a + b)$

$= \lim_{x \to 2} \frac{\sqrt{3-x} - 1}{(2-x)} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1}$

$= \lim_{x \to 2} \frac{(3-x) - 1}{(2-x)(\sqrt{3-x} + 1)}$

$= \lim_{x \to 2} \frac{2-x}{2-x(\sqrt{3-x} + 1)}$; $x \neq 2$

$= \frac{1}{\sqrt{3-2} + 1} = \frac{1}{\sqrt{1} + 1} = \frac{1}{1+1} = \frac{1}{2}$

Q.10 : $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \frac{\sqrt{1+0} - \sqrt{1-0}}{0} = \frac{0}{0}$

$= \lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

$= \lim_{x \to 0} \frac{(1+x) - (1-x)}{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}$

$= \lim_{x \to 0} \frac{2x}{x\left(\sqrt{1+x} + \sqrt{1-x}\right)} \; ; x \neq 0$

$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{1+1} = 1$

Q.11: $\lim_{x\to 1} \dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = \dfrac{(2\cdot 1-3)(\sqrt{1}-1)}{2(1)^2+1-3} = \dfrac{(-1)(1-1)}{2+1-3} = \dfrac{0}{0}$

$= \lim_{x\to 1} \dfrac{(2x-3)(\sqrt{x}-1)}{(2x^2+3x-2x-3)} \times \dfrac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$= \lim_{x\to 1} \dfrac{(2x-3)(x-1)}{x(2x+3)-1(2x+3)(\sqrt{x}+1)}$

$= \lim_{x\to 1} \dfrac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)} \quad ; x \neq 1$

$= \lim_{x\to 1} \dfrac{2x-3}{(2x+3)(\sqrt{x}+1)}$

$= \dfrac{2(1)-3}{(2(1)+3)(\sqrt{1}+1)} = \dfrac{2-3}{(5)(2)} = \dfrac{-1}{10}$

$\frac{1}{1} = 1$

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

$\frac{1}{1000} = 0.001$

$\frac{1}{1000000} = 0.0000001$

$\frac{1}{\infty} = 0$

$\lim_{n \to \infty} \frac{1}{n} = 0$

$\frac{1}{\infty} = 0$

Type : II

$\frac{\infty}{\infty}$

∞ : Infinite (अनंत)

Q.12: $\lim_{x \to \infty} \dfrac{4x^2 + 3x - 11}{3x^2 + 7x - 18}$ $\dfrac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^2 \left[4 + \frac{3}{x} - \frac{11}{x^2}\right]}{x^2 \left[3 + \frac{7}{x} - \frac{18}{x^2}\right]}$$

$$= \lim_{x \to \infty} \frac{4 + \frac{3}{x} - \frac{11}{x^2}}{3 + \frac{7}{x} - \frac{18}{x^2}}$$

$\frac{3}{x} \to 0$, $\frac{11}{x^2} \to 0$, $\frac{7}{x} \to 0$, $\frac{18}{x^2} \to 0$

$$= \frac{4 + 0 - 0}{3 + 0 - 0} = \frac{4}{3}$$

Q.13: $\lim_{x \to \infty} \frac{9x^2 - 2x - 7}{6x^2 + 5x + 1}$ $\frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^2 \left[9 - \frac{2}{x} - \frac{7}{x^2}\right]}{x^2 \left[6 + \frac{5}{x} + \frac{1}{x^2}\right]}$$

$$= \lim_{x \to \infty} \frac{9 - \frac{2}{x} - \frac{7}{x^2}}{6 + \frac{5}{x} + \frac{1}{x^2}}$$

$$= \frac{9 - \frac{2}{\infty} - \frac{7}{\infty^2}}{6 + \frac{5}{\infty} + \frac{1}{\infty^2}} = \frac{9 - 0 - 0}{6 + 0 + 0} = \frac{9}{6} = \frac{3}{2}$$

Q.14 : $\lim_{x \to \infty} \frac{9x^2 + 3x + 7}{5x^2 + 2x + 1} = \frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^2\left[9 + \frac{3}{x} + \frac{7}{x^2}\right]}{x^2\left[5 + \frac{2}{x} + \frac{1}{x^2}\right]}$$

$$= \lim_{x \to \infty} \frac{9 + \frac{3}{x} + \frac{7}{x^2}}{5 + \frac{2}{x} + \frac{1}{x^2}}$$

$$= \frac{9}{5}$$

Q.15: $\lim_{x \to \infty} \left| \frac{x^3 + 3x^2 + 6x + 5}{x^3 + 2x + 6} \right| = \frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^3 \left[1 + \frac{3}{x} + \frac{6}{x^2} + \frac{5}{x^3} \right]}{x^3 \left[1 + \frac{2}{x^2} + \frac{6}{x^3} \right]}$$

$$= \lim_{x \to \infty} \frac{1 + \frac{3}{x} + \frac{6}{x^2} + \frac{5}{x^3}}{1 + \frac{2}{x^2} + \frac{6}{x^3}}$$

$$= \frac{1}{1}$$

$$= 1$$

Q.16: $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x^3 - 2x^2 + 4x + 1} = \frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^2 \left[1 + \frac{5}{x} + \frac{6}{x^2}\right]}{x^3 \left[1 - \frac{2}{x} + \frac{4}{x^2} + \frac{1}{x^3}\right]}$$

$$= \lim_{x \to \infty} \frac{1}{x} \left\{ \frac{\left[1 + \frac{5}{x} + \frac{6}{x^2}\right]}{\left[1 - \frac{2}{x} + \frac{4}{x^2} + \frac{1}{x3}\right]} \right\}$$

$$= \frac{1}{\infty} \left[\frac{1 + 0 + 0}{1 - 0 + 0 + 0} \right]$$

$$= 0$$

Q.17: $\lim_{x \to \infty} \frac{x^3 + 3x^2 + x + 6}{x^2 + 5x + 6}$ $\frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{x^3 \left[1 + \frac{3}{x} + \frac{1}{x^2} + \frac{6}{x^3}\right]}{x^2 \left[1 + \frac{5}{x} + \frac{6}{x^2}\right]}$$

$$= \lim_{x \to \infty} \frac{x \left[1 + \frac{3}{x} + \frac{1}{x} + \frac{6}{x^3}\right]}{\left[1 + \frac{5}{x} + \frac{6}{x^2}\right]}$$

$= \infty$ (Undefined)

n प्राकृतिक संख्याओं के वर्गों का योग

Sum of squares of n Natural numbers.

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + 100^2 = \frac{100(100+1)(2(100)+1)}{6}$$

$$= \frac{100(101)(201)}{6}$$

$$= 338350$$

Q.18: $\lim_{n\to\infty} \dfrac{1^2+2^2+3^2+\text{----}+n^2}{n^3}$ $\dfrac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} \frac{(n^2+n)(2n+1)}{6n^3}$$

$$= \lim_{n\to\infty} \frac{2n^3+n^2+2n^2+n}{6n^3} =$$

$$= \lim_{n\to\infty} \frac{n^3\left[2+\frac{3}{n}+\frac{1}{n^2}\right]}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

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