Topic(s)

Previous Years Questions and practice problems Part 4

Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)

$$\int \cos ax \, dx = \frac{\sin ax}{a} + c$$

$$\int \sin ax \, dx = -\frac{\cos ax}{a} + c$$

Review:

$$\cos 2p = 1 - 2\sin^2 p$$

$$2\sin^2 p = 1 - \cos 2p$$

$$\sin^2 p = \frac{1-\cos 2p}{2} \quad \text{———} \; (1)$$

$$p = x; \; \sin^2 x = \frac{1-\cos 2x}{2}$$

$$p = 2x; \; \sin^2 2x = \frac{1-\cos 2(2x)}{2} = \frac{1-\cos 4x}{2}$$

$$p = \frac{x}{2}; \; \sin^2 \frac{x}{2} = \frac{1-\cos 2\frac{x}{2}}{2} = \frac{1-\cos x}{2}$$

⋮

Summary:

$$\sin^2 p = \frac{1-\cos 2p}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin^2 2x = \frac{1-\cos 4x}{2}$$

$$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$$

$$\cos 2p = 2\cos^2 p - 1$$

$$\cos 2p + 1 = 2\cos^2 p$$

$$\frac{\cos 2p + 1}{2} = \cos^2 p$$

$$\cos^2 p = \frac{1 + \cos 2p}{2} \quad \text{——— (2)}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\cos^2 2x = \frac{1 + \cos 4x}{2}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

Summary:

$$\sin^2 p = \frac{1-\cos 2p}{2}$$

$$\cos^2 p = \frac{1+\cos 2p}{2}$$

UPBTE 2018

Q. $I = \int \sin^2 2x \, dx$

$$= \int \left[\frac{1 - \cos 4x}{2} \right] dx$$

$$= \frac{1}{2} \left[1 \int dx - \int \cos 4x \, dx \right]$$

$$= \frac{1}{2} \left[x - \frac{\sin 4x}{4} \right] + c$$

Ans.

Q. $I = \int \sin^2 \frac{x}{2} \, dx$

$$= \int \left(\frac{1 - \cos x}{2} \right) dx$$

$$= \frac{1}{2} \left[1 \int dx - \int \cos x \, dx \right]$$

$$= \frac{1}{2} \left[x - \sin x \right] + c$$

Q. $I = \int \cos^2 x \, dx$

$\cos 2x = 2\cos^2 x - 1$

Solution:

$$= \int \left[\frac{1 + \cos 2x}{2}\right] dx$$

$$= \frac{1}{2}\left[1\int dx + \int \cos 2x \, dx\right]$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c \quad \text{Ans.}$$

Q. $I = \int \sin^2 x \, dx$

$$I = \int \frac{1 - \cos 2x}{2} \, dx$$

$$= \frac{1}{2} \int (1 - \cos 2x) \, dx$$

$$= \frac{1}{2} \left[\int 1 \, dx - \int \cos 2x \, dx \right]$$

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] + c$$

Ans

UPBTE2023

Q. $I = \int \sin^3 x \, dx$

$\sin 3x = 3\sin x - 4\sin^3 x$

$4\sin^3 x = 3\sin x - \sin 3x$

$\sin^3 x = \dfrac{3\sin x - \sin 3x}{4}$

$I = \int \dfrac{3\sin x - \sin 3x}{4} dx$

$= \dfrac{1}{4}\left[3\int \sin x \, dx - \int \sin 3x \, dx\right]$

$= \dfrac{1}{4}\left[3(-\cos x) - \left(-\dfrac{\cos 3x}{3}\right)\right] + c$

$= \dfrac{1}{4}\left[-3\cos x + \dfrac{\cos 3x}{3}\right] + c$

$= \dfrac{1}{4}\left[\dfrac{\cos 3x}{3} - 3\cos x\right] + c$ Ans

Q. $I = \int \cos^3 x \, dx$

$$= \int \left(\frac{\cos 3x + 3\cos x}{4} \right) dx$$

$$= \frac{1}{4} \int (\cos 3x + 3\cos x) \, dx$$

$$= \frac{1}{4} \left[\int \cos 3x \, dx + 3 \int \cos x \, dx \right]$$

$$= \frac{1}{4} \left[\frac{\sin 3x}{3} + 3 \sin x \right] + c$$

Ans

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\cos 3x + 3\cos x = 4\cos^3 x$$

$$\frac{\cos 3x + 3\cos x}{4} = \cos^3 x$$

UPBTE 2005

$\cos 2x = 2\cos^2 x - 1$

Q. $I = \int \cos^4 x \, dx$

$= \int (\cos^2 x)^2 \, dx$

$= \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$= \frac{1}{4}\left[\int \left(1^2 + \cos^2 2x + 2(1)\cos 2x\right) dx\right]$

$= \frac{1}{4}\left[\int 1\,dx + \int \cos^2 2x\,dx + 2\int \cos 2x\,dx\right]$

$= \frac{1}{4}\left[x + \int\left(\frac{1+\cos 4x}{2}\right)dx + 2\,\frac{\sin 2x}{2}\right] + c$

$= \frac{1}{4}\left[x + \frac{1}{2}\left\{\int 1\,dx + \int \cos 4x\,dx\right\} + \sin 2x\right] + c$

$$= \frac{1}{4}\left[x + \frac{1}{2}\left\{ \int 1\,dx + \int \cos 4x\,dx \right\} + \sin 2x \right] + c$$

$$= \frac{1}{4}\left[x + \frac{1}{2}\left\{ x + \frac{\sin 4x}{4} \right\} + \sin 2x \right] + c$$

$$= \frac{1}{4}\left[x + \frac{1}{2}x + \frac{\sin 4x}{8} + \sin 2x \right] + c$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{\sin 4x}{8} + \sin 2x \right] + c$$

Q. $I = \int \sin^4 x \, dx$

$$I = \int (\sin^2 x)^2 \, dx$$

$$= \int \left(\frac{1 - \cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4} \int (1^2 + \cos^2 2x - 2(1)\cos 2x) \, dx$$

$$= \frac{1}{4}\left[\int \left(1 + \frac{1 + \cos 4x}{2} - 2\cos 2x\right) dx \right]$$

$$= \frac{1}{4}\left[1\int dx + \frac{1}{2}\int (1 + \cos 4x)\, dx - 2\int \cos 2x \, dx \right]$$

$$= \frac{1}{4}\left[1\int dx + \frac{1}{2}\left\{ 1\int dx + \int \cos 4x \, dx \right\} - 2\int \cos 2x \, dx \right]$$

$$= \frac{1}{4}\left[x + \frac{1}{2}\left\{ x + \frac{\sin 4x}{4} \right\} - 2\,\frac{\sin 2x}{2} \right] + C$$

$$I = \frac{1}{4}\left[x + \frac{1}{2}\left\{x + \frac{\sin 4x}{4}\right\} - 2\,\frac{\sin 2x}{2}\right] + c$$

$$= \frac{1}{4}\left[x + \frac{1}{2}x + \frac{\sin 4x}{8} - \sin 2x\right] + c$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{\sin 4x}{8} - \sin 2x\right] + c$$

Solution to previous HW Question:

$$\int \sin(7x+5)\,dx = -\frac{\cos(7x+5)}{7} + C$$

HW Question: $\int e^{3x+4}\,dx =$

Comment

$3x+4 = t$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

let $x = a\tan\theta$

Next Video: Integration by parts (खण्डश: समाकलन)

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