Topic(s): Previous Years Questions (PYQ) Part 2

Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)

1: $f(x)$

$f'(x)$

let $f(x) = t$ — — —

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UPBTE 2017

Q. $I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

let $\cos x + \sin x = t$

$$\frac{d}{dx}(\cos x + \sin x) = \frac{dt}{dx}$$

$$-\sin x + \cos x = \frac{dt}{dx}$$

$$(\cos x - \sin x)\, dx = dt$$

$$I = \int \frac{1}{t} dt$$

$$= \log |t| + c$$

$$= \log |\cos x + \sin x| + c$$

Ans

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UPBTE 2017 Method 2

Q. $I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

$1 = \frac{a}{a} = \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$

$I = \int \frac{\frac{1}{\cos x}(\cos x - \sin x)}{\frac{1}{\cos x}(\cos x + \sin x)} dx$

$I = \int \frac{1 - \tan x}{1 + \tan x} dx$

$I = \int \frac{\tan \pi/4 - \tan x}{1 + 1 \tan x} dx$

$I = \int \frac{\tan \pi/4 - \tan x}{1 + \tan \frac{\pi}{4} \tan x} dx$

$I = \int \tan\left(\frac{\pi}{4} - x\right) dx$

; $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

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$$I = \int \tan\left(\frac{\pi}{4} - x\right) dx$$

$$= \frac{-\log\left|\cos\left(\frac{\pi}{4} - x\right)\right|}{-1} + c$$

$$I = \log\left|\cos\left(\frac{\pi}{4} - x\right)\right| + c$$

; $\int \tan x \, dx = -\log|\cos x| + c$

; $\int \tan(ax+b) = \frac{-\log|\cos(ax+b)|}{a} + c$

Q.

$$I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

Method 1

$$= \log|\cos x + \sin x| + c$$

Method 2

$$\stackrel{?}{=} \log\left|\cos\left(\frac{\pi}{4} - x\right)\right| + c$$

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Q.

$$I = \int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx$$

Method 1

$$= \log\left|\cos x + \sin x\right| + C$$

Method 2

$$= \log\left|\cos\left(\frac{\pi}{4} - x\right)\right| + C$$

Method 2

$$\log\left|\cos\left(\frac{\pi}{4} - x\right)\right| + C = \log\left|\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right| + C \quad ; \quad \cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$= \log\left|\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right| + C$$

$$= \log\left|\frac{\cos x + \sin x}{\sqrt{2}}\right| + C$$

$$= \log\left|\cos x + \sin x\right| - \log|\sqrt{2}| + C \quad ; \quad \log\frac{m}{n} = \log m - \log n$$

$$= \log\left|\cos x + \sin x\right| + C_1$$

=Method 1 Answer

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UPBTE 2002,08...

Q. $I = \int \frac{(p + q \tan^{-1} x)^m}{1 + x^2} dx$

let $\tan^{-1} x = t$

$\frac{d}{dx} \tan^{-1} x = \frac{d}{dx} t$

$\frac{1}{1+x^2} dx = dt$

$I = \int (p+qt)^m dt$

$= \frac{(p+qt)^{m+1}}{(m+1)} \times \frac{1}{q} + C$

$= \frac{1}{q(m+1)} (p + q \tan^{-1} x)^{m+1} + C$ Ans

HW : let $p + q \tan^{-1} x = t$

$\int t^n dt = \frac{t^{n+1}}{n+1} + C \; ; \; n \neq -1$

$\int (at+b)^n = \frac{(at+b)^{n+1}}{(n+1)} \times \frac{1}{a} + C$

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UPBTE 2003

Q. $I = \int \frac{1}{x(1+\log x)}\, dx$

let $1+\log x = t$

$\frac{d}{dx}(1+\log x) = \frac{dt}{dx}$

$0 + \frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x}\, dx = dt$

$I = \int \frac{1}{t}\, dt$

$= \log|t| + C$

$= \log|1+\log x| + C$ Ans

Hw : let $\log x = t$

$\int \frac{1}{t+1}\, dt = \log|t+1|$

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UPBTE 2018

Q. $I = \int \frac{dx}{1+e^x}$

$= \int \frac{e^{-x}}{e^{-x}+e^{-x}e^{x}}\,dx$

$I = \int \frac{e^{-x}}{e^{-x}+1}\,dx$

let $e^{-x}+1 = t$

$\frac{d}{dx}(e^{-x}+1) = \frac{d}{dx}t$

$-e^{-x}+0 = \frac{dt}{dx}$

$-e^{-x}\,dx = dt$

$e^{-x}\,dx = -dt$

$I = \int \frac{1}{t}(-dt)$

$= -\int \frac{1}{t}\,dt$

$= -\log|t| + c$

$= -\log|e^{-x}+1| + c$ Ans

$\frac{d}{dx}e^{ax} = ae^{ax}$

$\frac{d}{dx}e^{-1x} = \frac{d}{dx}e^{-x} = -1e^{-x}$

$= -e^{-x}$

$\frac{d}{dx}e^{-x} = -e^{-x}$

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UPBTE 2022

Q. $I = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

$\because \int \frac{f'(x)}{f(x)}\, dx = \log|f(x)|$

$= \log|e^x + e^{-x}| + c$

let $e^x + e^{-x} = t$

$\frac{d}{dx}(e^x + e^{-x}) = \frac{dt}{dx}$

$(e^x - e^{-x})\, dx = dt$

$I = \int \frac{1}{t}\, dt$

$= \log|t| + c$

$= \log|e^x + e^{-x}| + c$ Ans

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UPBTE 2019

Q. $I = \int \frac{\sin x}{1+\cos^2 x}\, dx$

let $\cos x = t$

$\frac{d}{dx}\cos x = \frac{d}{dx} t$

$-\sin x = \frac{dt}{dx}$

$-\sin x\, dx = dt$

$\sin x\, dx = -dt$

$I = \int \frac{1}{1+t^2}(-dt) = -\int \frac{1}{1+t^2} dt$

$= -\tan^{-1}(t) + c$

$= -\tan^{-1}(\cos x) + c$

$\frac{d}{dx}\cos x = -\sin x$

$\Rightarrow \alpha \sin x$

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Q. $I = \int \frac{x^7}{1+x^{16}}\,dx$

$\frac{d}{dx} x^n = n x^{n-1}$

$\frac{d}{dx} x^8 = 8 x^7$

$I = \int \frac{x^7}{1+(x^8)^2}\,dx$

let $x^8 = t$

$8x^7 dx = dt$

$x^7 dx = \frac{dt}{8}$

$\int \frac{1}{1+(t)^2} \frac{dt}{8} = \frac{1}{8}\int \frac{1}{1+t^2}\,dt$

$= \frac{1}{8} \tan^{-1}(t) + c$

$= \frac{1}{8} \tan^{-1}(x^8) + c$

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Solution to previous HW Question:

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

HW Question:

$$\int \tan x\,dx = ?$$

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Next video: Previous Years Questions Part 3

Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)

(1) $$\int f(x)\,dx = F(x) + c$$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

(2) More problems.....

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