Topic(s) :

Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)

$$\int f(x)\,dx = F(x) + C$$

- $\int f(x)\,dx$: Integrand (समाकल्य)
- $F(x)$: Integral (समाकल)
- C : Constant of Integration (समाकलन अचर)

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Q.1: $\int \cos(2x+3)\,dx = ?$

$\int \cos x\,dx = \sin x + c$

let $2x+3 = t$

$\frac{d}{dx}(2x+3) = \frac{dt}{dx}$

$2\frac{dx}{dx} + \frac{d}{dx}3 = \frac{dt}{dx}$

$2(1) + 0 = \frac{dt}{dx}$

$2\,dx = dt$

$dx = \frac{dt}{2}$

$\int \cos(2x+3) = \frac{\sin(2x+3)}{2} + c$

$\int \cos t \frac{dt}{2} = \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t + c = \frac{1}{2}\sin(2x+3) + c$

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$$\int e^x dx = e^x + c$$

Q.2: $\int e^{3x+4} dx = ?$

let $3x + 4 = t$

$\frac{d}{dx}(3x+4) = \frac{dt}{dx}$

$\frac{d}{dx}3x + \frac{d}{dx}4 = \frac{dt}{dx}$

$3(1) + 0 = \frac{dt}{dx}$

$3 = \frac{dt}{dx}$

$dx = \frac{dt}{3}$

$\int e^{3x+4} dx = \int e^t \frac{dt}{3}$

$= \frac{1}{3}\int e^t dt$

$= \frac{1}{3} e^t + c$

$= \frac{1}{3} e^{3x+4} + c$

$\int e^{3x+4} dx = \frac{e^{3x+4}}{3} + c$

If(यदि) $\int f(x)\,dx = F(x) + c$ then Prove that (तो सिद्ध कीजिए की)

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

Solution:

let $ax + b = t$

$$\frac{d}{dx}(ax+b) = \frac{dt}{dx}$$

$$a\frac{dx}{dx} + \frac{db}{dx} = \frac{dt}{dx}$$

$$a(1) + 0 = \frac{dt}{dx}$$

$$a\,dx = dt$$

$$dx = \frac{dt}{a}$$

$$\int f(ax+b)\,dx$$

$$= \int f(t)\,\frac{dt}{a}$$

$$= \frac{1}{a}\int f(t)\,dt$$

$$= \frac{1}{a}F(t) + c$$

$$= \frac{1}{a}F(ax+b) + c$$

$$\int f(x)\,dx = F(x) + c$$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

Examples:

1: $\int \cos x\,dx = \sin x + c$

$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$

$\int \cos(2x+3) = \frac{\sin(2x+3)}{2} + c$

$b = 0$

$\int \cos(ax+0)\,dx = \frac{\sin(ax+0)}{a} + c$

$\int \cos ax\,dx = \frac{\sin ax}{a} + c$

2: $\int e^x\,dx = e^x + c$

$\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$

$\int e^{3x+4}\,dx = \frac{e^{3x+4}}{3} + c$

$$\int f(x)\,dx = F(x) + c$$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

3: $\int \frac{1}{x}\,dx = \log|x| + c$

$$\int \frac{1}{ax+b} = \frac{\log|ax+b|}{a} + c$$

$$\int \frac{1}{2x+3}\,dx = \frac{\log|2x+3|}{2} + c$$

$$\int \frac{1}{1+x}\,dx = \frac{\log|1+x|}{1} + c$$

$$\int \frac{1}{1-x}\,dx = \frac{\log|1-x|}{-1} + c = -\log|1-x| + c$$

4: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad ; n \neq -1$

$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int (x+2)^3 \, dx = \frac{(x+2)^{3+1}}{1(3+1)} + c = \frac{(x+2)^4}{4} + c$

$\int \frac{1}{(x-2)^2} \, dx = \int (x-2)^{-2} \, dx = \frac{(x-2)^{-2+1}}{1(-2+1)} + c = \frac{(x-2)^{-1}}{-1}$

$= -\frac{1}{(x-2)} + c$

$\int \sqrt{1-5x} \, dx = \int (1-5x)^{1/2} \, dx = \frac{(1-5x)^{\frac{1}{2}+1}}{(\frac{1}{2}+1)(-5)} + c$

Standard Results: (मानक परिणाम)

(1): $\int \cos ax \, dx = \frac{\sin ax}{a} + c$

(2): $\int \sin ax \, dx = -\frac{\cos ax}{a} + c$

(3): $\int \sec^2 ax \, dx = \frac{\tan ax}{a} + c$

(4): $\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$

$f(x)$: Function of x

$\frac{d}{dx} f(x) = f'(x)$: Differential Coefficient of $f(x)$ with respect to x .

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$\int \frac{f'(x)}{f(x)} dx = ?$ Where $f'(x) = \frac{d}{dx} f(x)$

$f(x) = t$

$\frac{d}{dx} f(x) = \frac{dt}{dx}$

$f'(x)dx = dt$

$\int \frac{1}{t} dt = \log|t| + C$

$= \log|f(x)| + C$

$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

$$\int \frac{f'(x)}{[f(x)]^n}\,dx$$

let $f(x) = t$

$$\int \frac{f'(x)}{[f(x)]^n}\,dx = \frac{[f(x)]^{-n+1}}{(-n+1)} + c$$

$$\int [f(x)]^n f'(x)\,dx$$

let $f(x) = t$

$$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{(n+1)} + c$$

Summary:

$f(x)$

$f'(x)$

let $f(x) = t$ — — —

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Q. $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + C$

$$\int \frac{\sin x}{\cos x} \, dx$$

let $\cos x = t$

$$\frac{d}{dx}\cos x = \frac{d}{dx} t$$

$$-\sin x = \frac{dt}{dx}$$

$$-\sin x \, dx = dt$$

$$\sin x \, dx = -dt$$

$$\int \frac{1}{t}(-dt) = -\int \frac{1}{t} dt = -\log|t| + c$$

$$= -\log|\cos x| + C$$

Q: $\int \cot x \, dx = ?$

Solution:

$$I = \int \cot x \, dx$$

$$I = \int \frac{\cos x}{\sin x} \, dx$$

let $\sin x = t$

$$\frac{d}{dx} \sin x = \frac{dt}{dx}$$

$$\cos x = \frac{dt}{dx}$$

$$\cos x \, dx = dt$$

$$I = \int \frac{1}{t} \, dt$$

$$= \log |t| + c$$

$$= \log |\sin x| + c$$

$$\int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + c$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \log |\sin x| + c$$

$$\boxed{\int \cot x \, dx = \log |\sin x| + c}$$

$$\int \sec x \, dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$$

$$\int \operatorname{Cosec} x \, dx = \log|\operatorname{Cosec} x - \cot x| + c = \log\left|\tan\left(\frac{x}{2}\right)\right| + c$$

Summary:

1: $\int f(x)\,dx = F(x) + c$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

2: $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

3: $\int \tan x\,dx = -\log|\cos x| + c = \log|\sec x| + c$

4: $\int \cot x\,dx = \log|\sin x| + c$

5: $\int \sec x\,dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$

6: $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c = \log\left|\tan\left(\frac{x}{2}\right)\right| + c$

HW Question: $\int \sin x\,dx = ?$

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