Topic(s) : Previous Years Questions (PYQ) Part 5

Chapter 3: Integration by parts (खण्डशः समाकलन)

(1) $\int u\,v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\cdot\int v\,dx\right\}dx$

(2) I L A T E Rule

(3) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

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Q. $I = \int \sin^{-1}x \, dx$

$\int \log x \, dx = \int \log x \times 1 \, dx$

$I = \int \sin^{-1}x \times 1 \, dx$

$= \sin^{-1}x \int 1 \, dx - \int \left\{ \frac{d}{dx} \sin^{-1}x \cdot \int 1 \, dx \right\} dx$

$= \sin^{-1}x \; x - \int \left\{ \frac{1}{\sqrt{1-x^2}} \; x \right\} dx$

$I = x \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} \, dx$

$I = x \sin^{-1}x - I_1 \quad \text{---} (1) \quad I_1 = \int \frac{x}{\sqrt{1-x^2}} \, dx$

ILATE Rule

$$I_1 = \int \frac{x}{\sqrt{1-x^2}}\, dx$$

let $1 - x^2 = t$

$$\frac{d}{dx}(1-x^2) = \frac{dt}{dx}$$

$$0 - 2x = \frac{dt}{dx}$$

$$-2x\, dx = dt$$

$$x\, dx = \frac{-dt}{2}$$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}}\, dx = \int \frac{1}{\sqrt{t}} \left(\frac{-dt}{2}\right)$$

$$I_1 = -\frac{1}{2} \int t^{-1/2}\, dt$$

$$I_1 = -\frac{1}{2} \int t^{-1/2}\, dt$$

$$= -\frac{1}{2} \frac{t^{-1/2+1}}{-\frac{1}{2}+1} + C$$

$$= -\frac{1}{2} \frac{t^{1/2}}{\frac{1}{2}} + C$$

$$= -\frac{1}{2} \times \frac{2}{1} \sqrt{t} + C$$

$$I_1 = -\sqrt{1-x^2} + C$$

I_1 का मान समी० (1) में रखने पर

$$I = x \sin^{-1} x - I_1$$

$$= x \sin^{-1} x - \left[-\sqrt{1-x^2} + c\right]$$

$$= x \sin^{-1} x + \sqrt{1-x^2} - c$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + c_1 ; \quad c_1 = -c$$

Q. $\int \cos^{-1} x \, dx$

$I = \int \cos^{-1} x \times 1 \, dx$

$I = \cos^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} \cos^{-1} x \cdot \int 1 \, dx \right\} dx$

$I = \cos^{-1} x \; x - \int \left\{ \frac{-1}{\sqrt{1-x^2}} \cdot x \right\} dx$

$I = \cos^{-1} x \; x + \int \frac{x}{\sqrt{1-x^2}} dx$

$I = x \cos^{-1} x + I_1 \quad \text{---} (1)$ where $I_1 = \int \frac{x}{\sqrt{1-x^2}} dx$

$I_1 = \int \frac{x}{\sqrt{1-x^2}} dx$

ILATE Rule

$$I_1 = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

let $1-x^2 = t$

$$\frac{d}{dx}(1-x^2) = \frac{d}{dx}t$$

$$0 - 2x = \frac{d}{dx}t$$

$$-2x\,dx = dt$$

$$x\,dx = \frac{-dt}{2}$$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{t}}\left(\frac{-dt}{2}\right)$$

$$= -\frac{1}{2}\int t^{-1/2}\,dt = -\frac{1}{2}\,\frac{t^{-1/2+1}}{-1/2+1} + C$$

$$I_1 = -\frac{1}{2}(2)\,t^{1/2} + C$$

$$I_1 = -\,t^{1/2} + C$$

$$I_1 = -\sqrt{t} + C$$

$$I_1 = -\sqrt{1-x^2} + C$$

समीकरण (1) में I_1 का मान रखनें पर

$$I = x\cos^{-1}x + I_1 \quad \text{——} \quad (1)$$

$$I = x\cos^{-1}x + \left(-\sqrt{1-x^2} + C\right)$$

$$I = \int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2} + C$$

Ans

UPBTE

Q. $\int x \tan^{-1} x \, dx$

$I = \int \tan^{-1} x \; x \, dx$

$I = \tan^{-1} x \int x \, dx - \int \left\{ \frac{d}{dx} \tan^{-1} x \cdot \int x \, dx \right\} dx$

$= \tan^{-1} x \; \frac{x^2}{2} - \int \left\{ \frac{1}{1+x^2} \cdot \frac{x^2}{2} \right\} dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2+1-1}{1+x^2} \, dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[\int \left[\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right] dx \right]$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[\int 1 \, dx - \int \frac{1}{1+x^2} dx \right]$

ILATE Rule

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1\,dx - \int \frac{1}{1+x^2}\,dx\right]$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[x - \tan^{-1}x\right] + C$$

$$\int x\tan^{-1}x\,dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$$

Ans

UPBTE 2015

Q. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

$\int \frac{f'(x)}{f(x)} dx = \log|f(x)|$

$= \int 2\tan^{-1}x \, dx$

$= 2\int \tan^{-1}x \times 1 \, dx$

$= 2\left[\tan^{-1}x \int 1\,dx - \int \left\{\frac{d}{dx}\tan^{-1}x \cdot \int 1\,dx\right\}dx\right]$

$= 2\left[\tan^{-1}x \cdot x - \int \left\{\frac{1}{1+x^2} \cdot x\right\}dx\right]$

$= 2\left[x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2} dx\right]$

$= 2\left[x\tan^{-1}x - \frac{1}{2}\log|1+x^2|\right] + c$

Q. $\int x \log x \, dx$

I L A T E

A L

$I = \int \log x \cdot x \, dx$

$= \log x \int x \, dx - \int \left\{ \frac{d}{dx} \log x \cdot \int x \, dx \right\} dx$

$= \log x \cdot \frac{x^2}{2} - \int \left\{ \frac{1}{x} \cdot \frac{x^2}{2} \right\} dx$

$= \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$= \frac{x^2}{2} \log x - \frac{x^2}{4} + C$

Solution to previous practice problem:

$\cos 2A = 2\cos^2 A - 1$

$\cos x = \cos\left(2\frac{x}{2}\right) = 2\cos^2\frac{x}{2} - 1$

$$\int \sqrt{1+\cos x}\, dx = \int \sqrt{1+2\cos^2\frac{x}{2}-1}\, dx$$

$$= \int \sqrt{2\cos^2\frac{x}{2}}\, dx$$

$$= \int \sqrt{2}\sqrt{\cos^2\frac{x}{2}}\, dx$$

$$= \sqrt{2}\int \cos\frac{x}{2}\, dx$$

$$= \sqrt{2}\ \frac{\sin\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\sqrt{2}\sin\frac{x}{2} + c$$

Ans

Practice Problem: $\int \sin^2 2x\, dx = ?$

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