Topic(s) : Previous Years Questions (PYQ) Part 4

Chapter 3: Integration by parts (खण्डशः समाकलन)

$$\int u\,v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\cdot\int v\,dx\right\}dx$$

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Review

$$\frac{d}{dx}x^n = n\,x^{n-1}$$

(1) $$\frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{d}{dx}(x+1)^{-1} = \frac{d(x+1)^{-1}}{d(x+1)}\frac{d(x+1)}{dx}$$

$$= -1\,(x+1)^{-1-1}\,(1+0)$$

$$= -(x+1)^{-2}$$

$$= \boxed{\frac{-1}{(x+1)^2}}$$

(2) $$\int e^x\,dx = e^x$$

UPBTE 2022..

Q. $\int \frac{x\, e^x}{(x+1)^2}\, dx$

$$I = \int \frac{(x+1-1)\, e^x}{(x+1)^2}\, dx$$

$$= \int \frac{(x+1)\, e^x - e^x}{(x+1)^2}\, dx \qquad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$= \int \left\{ \frac{(x+1)\, e^x}{(x+1)^2} - \frac{e^x}{(x+1)^2} \right\} dx$$

$$= \int \frac{e^x}{(x+1)}\, dx - \int \frac{e^x}{(x+1)^2}\, dx$$

$$= \int (x+1)^{-1} e^x\, dx - \int \frac{e^x}{(x+1)^2}\, dx$$

$$= (x+1)^{-1} \int e^x\, dx - \int \left\{ \frac{d}{dx}(x+1)^{-1} \cdot \int e^x\, dx \right\} dx - \int \frac{e^x}{(x+1)^2}\, dx$$

$$= (x+1)^{-1} \int e^x\,dx - \int \left\{ \frac{d}{dx}(x+1)^{-1} \cdot \int e^x\,dx \right\} dx - \int \frac{e^x}{(x+1)^2}\,dx$$

$$= (x+1)^{-1} e^x - \int \left\{ \frac{-1}{(x+1)^2} \cdot e^x \right\} dx - \int \frac{e^x}{(x+1)^2}\,dx$$

$$= (x+1)^{-1} e^x + \int \frac{e^x}{(x+1)^2}\,dx - \int \frac{e^x}{(x+1)^2}\,dx$$

$$= \frac{e^x}{(x+1)} + C$$

$$\int \frac{x\,e^x}{(x+1)^2}\,dx = \frac{e^x}{(x+1)} + C$$

Ans

UPBTE 2006..

Q. $\int \frac{e^x(x^2+1)}{(x+1)^2}\,dx$

$$I = \int \frac{e^x\,(x^2+1+2x-2x)}{(x+1)^2}\,dx$$

$$I = \int \frac{e^x\,\{(x+1)^2-2x\}}{(x+1)^2}\,dx$$

$$= \int \left\{\frac{e^x(x+1)^2}{(x+1)^2} - \frac{2xe^x}{(x+1)^2}\right\} dx$$

$$= \int \frac{e^x(x+1)^2}{(x+1)^2}\,dx - 2\int \frac{xe^x}{(x+1)^2}\,dx$$

$$= \int e^x\,dx - 2\int \frac{xe^x}{(x+1)^2}\,dx$$

$$= e^x - 2\frac{e^x}{x+1} + c$$

Q. $\int e^{ax} \cos bx \, dx$

ILATE

$I = \int \cos bx \; e^{ax} \, dx$

$I = \cos bx \cdot \int e^{ax} dx - \int \left\{ \frac{d}{dx} \cos bx \int e^{ax} dx \right\} dx$

$= \cos bx \; \frac{e^{ax}}{a} - \int \left\{ -b \sin bx \; \frac{e^{ax}}{a} \right\} dx$

$= \cos bx \; \frac{e^{ax}}{a} + \frac{b}{a} \int \sin bx \; e^{ax} \, dx$

$= \cos bx \; \frac{e^{ax}}{a} + \frac{b}{a} \left\{ \sin bx \int e^{ax} dx - \int \left\{ \frac{d}{dx} \sin bx \int e^{ax} dx \right\} dx \right\}$

$= \cos bx \; \frac{e^{ax}}{a} + \frac{b}{a} \left\{ \sin bx \; \frac{e^{ax}}{a} - \int \left\{ b \cos bx \; \frac{e^{ax}}{a} \right\} dx \right\}$

$I = \cos bx \; \frac{e^{ax}}{a} + \frac{b}{a^2} \sin bx \, e^{ax} - \frac{b^2}{a^2} \int \cos bx \; e^{ax} \, dx$

$I = e^{ax} \left[\frac{\cos bx}{a} + \frac{b}{a^2} \sin bx \right] - \frac{b^2}{a^2} I + c$

$$I = e^{ax}\left[\frac{\cos bx}{a} + \frac{b}{a^2}\sin bx\right] - \frac{b^2}{a^2} I + c$$

$$I + \frac{b^2}{a^2} I = e^{ax}\left[\frac{\cos bx}{a} + \frac{b}{a^2}\sin bx\right] + c$$

$$I\left[1 + \frac{b^2}{a^2}\right] = e^{ax}\left[\frac{\cos bx}{a} + \frac{b}{a^2}\sin bx\right] + c$$

$$I\left[\frac{a^2+b^2}{a^2}\right] = e^{ax}\left[\frac{\cos bx}{a} + \frac{b}{a^2}\sin bx\right] + c$$

$$I = \frac{a^2}{a^2+b^2}\left\{e^{ax}\left[\frac{\cos bx}{a} + \frac{b}{a^2}\sin bx\right] + c\right\}$$

$$I = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right] + c_1$$

$$c_1 = \frac{a^2}{a^2+b^2} c$$

Q. $\int e^{ax} \sin bx \, dx$

$$I = \int \sin bx \, e^{ax} \, dx$$

$$I = \sin bx \int e^{ax} dx - \int \left\{ \frac{d}{dx}(\sin bx) \cdot \int e^{ax} dx \right\} dx$$

$$I = \sin bx \frac{e^{ax}}{a} - \int \left\{ b \cos bx \cdot \frac{e^{ax}}{a} \right\} dx$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a} \int \left\{ \cos bx \cdot e^{ax} \right\} dx$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a} \left[\cos bx \int e^{ax} dx - \int \left\{ \frac{d}{dx} \cos bx \int e^{ax} dx \right\} dx \right]$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a} \left[\cos bx \frac{e^{ax}}{a} - \int \left\{ -b \sin bx \frac{e^{ax}}{a} \right\} dx \right]$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a} \left[\cos bx \frac{e^{ax}}{a} + \frac{b}{a} \int \sin bx \, e^{ax} dx \right]$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a^2} \cos bx \, e^{ax} - \frac{b^2}{a^2} \int \sin bx \, e^{ax} dx$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a^2} \cos bx \, e^{ax} - \frac{b^2}{a^2} I + C$$

$$I = \sin bx \frac{e^{ax}}{a} - \frac{b}{a^2}\cos bx\ e^{ax} - \frac{b^2}{a^2} I + c$$

$$I + \frac{b^2}{a^2} I = e^{ax}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + c$$

$$I\left(1 + \frac{b^2}{a^2}\right) = e^{ax}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + c$$

$$I\left(\frac{a^2+b^2}{a^2}\right) = e^{ax}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + c$$

$$I = \frac{a^2}{a^2+b^2}\left[e^{ax}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + c\right]$$

$$I = \frac{a^2 e^{ax}}{a^2+b^2}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + \frac{a^2}{a^2+b^2} c$$

$$I = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + c_1 \qquad ; \ c_1 = \frac{a^2}{a^2+b^2} c$$

Ans

Summary: $\int e^{ax} \cos bx \, dx$

$\int e^{ax} \sin bx \, dx$

Q. $\int e^{2x} \sin 3x \, dx$

Q. $\int e^{x} \sin x \, dx$

Solution to previous practice problem:

$\sin(2A) = 2\sin A \cos A$

$\sin x = \sin\left(2\cdot\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$\int \sqrt{1+\sin x}\, dx = \int \sqrt{1 + 2\sin\frac{x}{2}\cos\frac{x}{2}}\, dx$$

$$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\, dx \qquad \sin^2\theta + \cos^2\theta = 1$$

$$= \int \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}\, dx$$

$$= \int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$$

$$= \int \sin\frac{x}{2}\, dx + \int \cos\frac{x}{2}\, dx$$

$$= -\frac{\cos\left(\frac{x}{2}\right)}{1/2} + \frac{\sin\left(\frac{x}{2}\right)}{1/2} + c$$

$$= -2\cos\frac{x}{2} + 2\sin\frac{x}{2} + c = 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$$

Ans

Practice Problem: $\int \sqrt{1+\cos x}\, dx =$

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