

Topic(s) :

Previous Years Questions (PYQ) Part 3

Chapter 3: Integration by parts (खण्डशः समाकलन)

$$(1) \int u v dx = u \int v dx - \int \left\{ \frac{du}{dx} \cdot \int v dx \right\} dx$$

(2) ILATE Rule

$$(3) \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

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Q.

$$\int \frac{x^2 \tan^{-1} x}{1+x^2} dx$$

$$\text{let } \tan^{-1} x = t \Rightarrow x = \tan t$$

$$\frac{d}{dx} \tan^{-1} x = \frac{dt}{dx} \quad x^2 = \tan^2 t$$

$$\frac{1}{1+x^2} dx = dt$$

$$\int \frac{x^2 \tan^{-1} x}{1+x^2} dx = \int \begin{matrix} \tan^2 t & t & dt \\ \downarrow & \downarrow & \\ T & A & \\ \text{I L A T E} & & \end{matrix}$$

$$\int t \tan^2 t dt = \int t (\sec^2 t - 1) dt$$

$$= \int (t \sec^2 t - t) dt$$

$$= \int t \sec^2 t dt - \int t dt$$

$$1 + \tan^2 t = \sec^2 t$$

$$\tan^2 t = \sec^2 t - 1$$

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$$\int t \sec^2 t \, dt - \int t \, dt$$

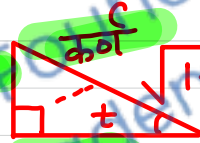
$$= t \int \sec^2 t \, dt - \int \left\{ \frac{dt}{dt} \cdot \int \sec^2 t \, dt \right\} dt - \frac{t^2}{2} + C$$

$$= t \tan t - \int \{1 \cdot \tan t\} dt - \frac{t^2}{2} + C$$

$$= t \tan t - \log |\sec t| - \frac{t^2}{2} + C$$

$$\tan^{-1} x = t \Rightarrow x = \tan t$$

$$\tan t = \frac{x}{1}$$



$$\sec t = \frac{\text{कोण}}{\text{आधक}} = \frac{\sqrt{1+x^2}}{1}$$

$$\begin{aligned} \text{कोण}^2 &= 1^2 + x^2 \\ \text{कोण} &= \sqrt{1+x^2} \end{aligned}$$

$$\begin{aligned} \int \frac{x^2 \tan^{-1} x}{1+x^2} dx &= \tan^{-1} x \cdot x - \log |\sqrt{1+x^2}| - \frac{1}{2} (\tan^{-1} x)^2 + C \\ &= x \tan^{-1} x - \log |\sqrt{1+x^2}| - \frac{1}{2} (\tan^{-1} x)^2 + C \end{aligned}$$

Q.

$$\int \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$\int \frac{x \tan^{-1} x}{(1+x^2)(1+x^2)^{\frac{1}{2}}} dx$$

$$\text{let } \tan^{-1} x = t \Rightarrow x = \tan t$$

$$\frac{d \tan^{-1} x}{dx} = \frac{dt}{dt} \Rightarrow x^2 = \tan^2 t$$

$$\frac{1}{1+x^2} dx = dt$$

$$\int \frac{x \tan^{-1} x}{(1+x^2)(1+x^2)^{\frac{1}{2}}} dx = \int \frac{\tan t \cdot t}{(1+\tan^2 t)^{\frac{1}{2}}} dt = \int \frac{\tan t \cdot t}{\sec t} dt$$

$$\int \frac{\tan t \cdot t}{\sec t} dt = \int \frac{\sin t \cdot t}{\cos t \sec t} dt$$

$$\sec t = \frac{1}{\cos t}$$

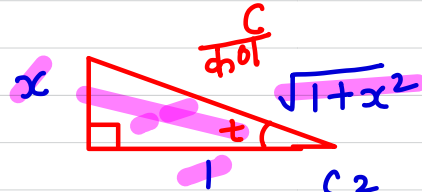
$$\cos t \sec t = 1$$

$$\int \frac{\sin t}{\cos t} dt = \int \underset{\substack{\downarrow \\ T}}{\sin t} \underset{\substack{\downarrow \\ A}}{t} dt$$

ILATE

$$\begin{aligned} \int t \sin t dt &= t \int \sin t dt - \int \left\{ \frac{dt}{dt} \cdot \int \sin t dt \right\} dt \\ &= t(-\cos t) - \int \{ 1 \cdot (-\cos t) \} dt \\ &= -t \cos t + \int \cos t dt \\ &= -t \cos t + \sin t + C \end{aligned}$$

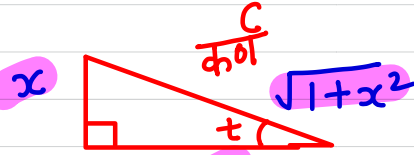
$$\begin{aligned} \tan^{-1} x = t &\Rightarrow x = \tan t \\ \tan t &= \frac{x}{1} \end{aligned}$$



$$\begin{aligned} \text{कोट}^2 &= 1^2 + x^2 \\ \text{कोट} &= \sqrt{1+x^2} \end{aligned}$$

$$= -t \cos t + \sin t + C$$

$$\tan^{-1} x = t \Rightarrow x = \tan t$$
$$\tan t = \frac{x}{1}$$



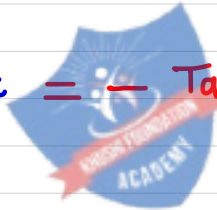
$$\text{hyp}^2 = 1^2 + x^2$$
$$\text{hyp} = \sqrt{1+x^2}$$

$$\cos t = \frac{1}{\sqrt{1+x^2}}$$

$$\sin t = \frac{x}{\sqrt{1+x^2}}$$

$$\rightarrow = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + C$$

$$\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + C$$



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