Topic(s): Previous Years Questions (PYQ) Part 2

Chapter 3: Integration by parts (खण्डशः समाकलन)

$$\int u\, v\, dx = u \int v\, dx - \int \left\{ \frac{du}{dx} \cdot \int v\, dx \right\} dx$$

ILATE Rule

Q. $\int e^{2x} \sin 3x \, dx$

$$I = \int \sin 3x \, e^{2x} \, dx \qquad \int u\, v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \cdot \int v \, dx \right\} dx$$

$$I = \sin 3x \int e^{2x} dx - \int \left\{ \frac{d}{dx} \sin 3x \cdot \int e^{2x} dx \right\} dx$$

$$I = \sin 3x \frac{e^{2x}}{2} - \int \left\{ 3 \cos 3x \frac{e^{2x}}{2} \right\} dx$$

$$I = \sin 3x \frac{e^{2x}}{2} - \frac{3}{2} \int \cos 3x \, e^{2x} \, dx$$

$$\frac{(-)(-)(-)}{+} \quad \frac{(-)}{-}$$

$$I = \sin 3x \frac{e^{2x}}{2} - \frac{3}{2} \left[\cos 3x \int e^{2x} dx - \int \left\{ \frac{d}{dx} \cos 3x \cdot \int e^{2x} dx \right\} dx \right]$$

$$I = \sin 3x \frac{e^{2x}}{2} - \frac{3}{2} \left[\cos 3x \frac{e^{2x}}{2} - \int \left\{ -3 \sin 3x \frac{e^{2x}}{2} \right\} dx \right]$$

$$I = \sin 3x \frac{e^{2x}}{2} - \frac{3}{4} \cos 3x \, e^{2x} - \frac{9}{4} \int \sin 3x \, e^{2x} \, dx$$

ILATE Rule

$$I = \sin 3x \, \frac{e^{2x}}{2} - \frac{3}{4} \cos 3x \, e^{2x} - \frac{9}{4} I$$

$$I + \frac{9}{4} I = e^{2x} \left[\frac{\sin 3x}{2} - \frac{3}{4} \cos 3x \right] + c$$

$$I \left[\frac{1}{1} + \frac{9}{4} \right] = e^{2x} \left[\frac{\sin 3x}{2} - \frac{3}{4} \cos 3x \right] + c$$

$$I \left[\frac{4+9}{4} \right] = e^{2x} \left[\frac{\sin 3x}{2} - \frac{3}{4} \cos 3x \right] + c$$

$$\frac{13}{4} I = e^{2x} \left[\frac{\sin 3x}{2} - \frac{3}{4} \cos 3x \right] + c$$

$$I = \frac{4}{13} \left\{ e^{2x} \left[\frac{\sin 3x}{2} - \frac{3}{4} \cos 3x \right] + c \right\}$$

$$I = \frac{4}{13}\left\{ e^{2x}\left[\frac{\sin 3x}{2} - \frac{3}{4}\cos 3x\right] + C \right\}$$

$$I = \frac{e^{2x}}{13}\left[2\sin 3x - 3\cos 3x\right] + c_1 \quad ; \quad c_1 = 4c$$

Ans

Q. $I = \int \sec^3 x \, dx$

$$\int u\, v\, dx = u \int v\, dx - \int \left\{ \frac{du}{dx} \cdot \int v\, dx \right\} dx$$

$$I = \int \sec x \sec^2 x \, dx$$

$$I = \sec x \int \sec^2 x \, dx - \int \left\{ \frac{d}{dx} \sec x \cdot \int \sec^2 x \, dx \right\} dx$$

$$I = \sec x \tan x - \int \{ \sec x \tan x \cdot \tan x \} dx$$

$$I = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$$I = \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$I = \sec x \tan x - \int (\sec^3 x - \sec x) \, dx$$

$$I = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$I = \sec x \tan x - I + \log(\sec x + \tan x) + c$$

ILATE Rule

$$I = \sec x \tan x - \cancel{I} + \log(\sec x + \tan x) + c$$

$$2I = \sec x \tan x + \log(\sec x + \tan x) + c$$

$$I = \frac{1}{2}\left[\sec x \tan x + \log(\sec x + \tan x\right] + c_1$$ **Ans**

$$c_1 = \frac{c}{2}$$

UPBTE 2023

Q.

$$I = \int x^2 \sin x \, dx$$

$$\int u\, v\, dx = u \int v\, dx - \int \left\{ \frac{du}{dx} \cdot \int v\, dx \right\} dx$$

$$I = x^2 \int \sin x \, dx - \int \left\{ \frac{d}{dx} x^2 \cdot \int \sin x \, dx \right\} dx$$

$$= x^2(-\cos x) - \int \left\{ 2x \cdot (-\cos x) \right\} dx$$

$$= -x^2 \cos x + 2 \int x \cos x \, dx$$

$$= -x^2 \cos x + 2 \left[x \int \cos x \, dx - \int \left\{ \frac{dx}{dx} \cdot \int \cos x \, dx \right\} dx \right]$$

$$= -x^2 \cos x + 2 \left[x \sin x - \int \left\{ 1 \cdot \sin x \right\} dx \right]$$

$$= -x^2 \cos x + 2 \left[x \sin x - \int \sin x \, dx \right]$$

$$= -x^2 \cos x + 2 \left[x \sin x + \cos x \right] + C$$

Ans

ILATE Rule

UPBTE 2008

Q. $\int x \cos^2 x \, dx$

$$\int u\, v\, dx = u \int v\, dx - \int \left\{ \frac{du}{dx} \cdot \int v\, dx \right\} dx$$

$$I = \int x \left[\frac{1 + \cos 2x}{2} \right] dx$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$I = \frac{1}{2} \left[\int (x + x \cos 2x)\, dx \right]$$

$$I = \frac{1}{2} \left[\int x\, dx + \int x \cos 2x\, dx \right]$$

$$= \frac{1}{2} \left[\frac{x^2}{2} + x \int \cos 2x\, dx - \int \left\{ \frac{d}{dx} x \cdot \int \cos 2x\, dx \right\} dx \right]$$

$$I = \frac{1}{2} \left[\frac{x^2}{2} + x \frac{\sin 2x}{2} - \int \left\{ 1 \cdot \frac{\sin 2x}{2} \right\} dx \right]$$

ILATE Rule

$$I = \frac{1}{2}\left[\frac{x^2}{2} + x\,\frac{\sin 2x}{2} - \int\left\{1 \cdot \frac{\sin 2x}{2}\right\}dx\right]$$

$$I = \frac{1}{2}\left[\frac{x^2}{2} + \frac{x\sin 2x}{2} - \frac{1}{2}\left(\frac{-\cos 2x}{2}\right)\right] + c$$

$$I = \frac{1}{2}\left[\frac{x^2}{2} + \frac{x\sin 2x}{2} + \frac{1}{4}\cos 2x\right] + c \quad \text{Ans}$$

Solution to previous practice problem:

$$\int \log x \, dx = \int \log x \times 1 \, dx = x[\log x - 1] + c$$

Practice Problem: $\int \sqrt{1 + \sin x} \, dx =$

COMMENT

Next video: Previous Years Questions (PYQ) Part 3

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