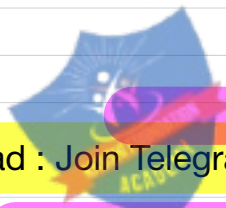


Chapter 4: Integration by Partial Fraction ( आंशिक भिन्नो द्वारा समाकलन )

Topic(s) :

Previous Years Questions ( PYQ) Part 4

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$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$$

$$\text{let } \tan x = t$$

$$\frac{d}{dx} \tan x = \frac{dt}{dx}$$

$$\sec^2 x dx = dt$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx = \int \frac{1}{(1+t)(2+t)} dt$$



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$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx = \int \frac{1}{(1+t)(2+t)} dt$$

$$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \quad \text{--- (1)}$$

$$1 = A(2+t) + B(1+t) \quad \text{--- (2)}$$

put  $1+t=0$  in eq. (2)  $1 = A(2-1) + B(0)$   
 $t = -1$   $A = 1$

put  $2+t=0$  in eq. (2)  $1 = A(0) + B(1-2)$   
 $t = -2$   $B = -1$

$$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx = \int \frac{1}{(1+t)(2+t)} dt = \int \left( \frac{1}{1+t} - \frac{1}{2+t} \right) dt$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx = \int \frac{1}{(1+t)(2+t)} dt = \int \left( \frac{1}{1+t} - \frac{1}{2+t} \right) dt$$

$$= \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$$

$$= \log|1+t| - \log|2+t| + C$$

$$= \log \left| \frac{1+t}{2+t} \right| + C$$

$$= \log \left| \frac{1+\tan x}{2+\tan x} \right| + C \quad \underline{\text{Ans}}$$

$$\int \frac{1}{ax+b} dx = \frac{\log|ax+b|}{a} + C$$



$$\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$$

let  $\sin x = t$

$$\frac{d}{dx} \sin x = \frac{dt}{dx}$$

$$\cos x dx = dt$$

$$\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx = \int \frac{1}{(1+t)(2+t)} dt$$



$$= \log \left| \frac{1+t}{2+t} \right| + C$$

$$= \log \left| \frac{1 + \sin x}{2 + \sin x} \right| + C \quad \underline{\text{Ans}}$$

$$\int \frac{dx}{x(x^n + 1)}$$

$$\text{let } x^n = t$$

$$\frac{d}{dx} x^n = \frac{dt}{dx}$$

$$n x^{n-1} dx = dt$$

$$\int \frac{dx}{x(x^n + 1)} = \int \frac{n x^{n-1} dx}{n x^{n-1} x (x^n + 1)} = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

$$\int \frac{dx}{x(x^n + 1)} = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

$$x^{n-1} x^1 = x^{n-1+1} = x^n = t$$

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$$\int \frac{dx}{x(x^n + 1)} = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \quad \text{--- (1)}$$

$$1 = A(t+1) + Bt \quad \text{--- (2)}$$

put  $t=0$  in eq. (2)

$$1 = A(0+1) + B(0)$$

put  $t+1=0$  in eq. (2)  
 $t=-1$

$$A = 1$$
$$1 = A(0) + B(-1)$$

$$B = -1$$

$$\frac{1}{t(t+1)} = \frac{1}{t} + \frac{-1}{t+1}$$

$$\int \frac{dx}{x(x^n + 1)} = \frac{1}{n} \int \frac{1}{t(t+1)} dt = \frac{1}{n} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$\begin{aligned}\int \frac{dx}{x(x^n+1)} &= \frac{1}{n} \int \frac{1}{t(t+1)} dt = \frac{1}{n} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt \\ &= \frac{1}{n} \left[ \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right] \\ &= \frac{1}{n} \left[ \log |t| - \log |t+1| \right] + C \\ &= \frac{1}{n} \log \left| \frac{t}{t+1} \right| + C \\ &= \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C\end{aligned}$$

Ans



$$x^3 x = x^4 = t$$

$$\int \frac{dx}{x(x^4+1)}$$

$$\text{let } x^4 = t$$

$$\frac{d x^4}{d x} = \frac{d t}{d x}$$

$$4 x^3 dx = dt$$

$$\int \frac{dx}{x(x^4+1)} = \int \frac{4 x^3 dx}{4 x^3 x(x^4+1)} = \frac{1}{4} \int \frac{1}{t(t+1)} dt$$

$$= \frac{1}{4} \log \left| \frac{x^4}{x^4+1} \right| + C$$



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Q.

$$\int \frac{dx}{1+3e^x+2e^{2x}}$$

$$e^{2x} = (e^x)^2$$

$$\text{let } e^x = t$$

$$\frac{d e^x}{dx} = \frac{d t}{dx}$$

$$e^x dx = dt$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{e^x dx}{e^x (1+3e^x+2(e^x)^2)} = \int \frac{dt}{t(1+3t+2t^2)}$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)} dt = \int \frac{1}{t(t+1)(2t+1)} dt$$

$$2t^2 + 3t + 1 = 2t^2 + 2t + t + 1 = 2t(t+1) + 1(t+1) \\ = (t+1)(2t+1)$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)} dt = \int \frac{1}{t(t+1)(2t+1)} dt$$

$$\frac{1}{t(t+1)(2t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{2t+1} \quad \text{--- (1)}$$

$$1 = A(t+1)(2t+1) + Bt(2t+1) + Ct(t+1) \quad \text{--- (2)}$$

$$A=1 \quad B=1 \quad C=-4$$

$$\frac{1}{t(t+1)(2t+1)} = \frac{1}{t} + \frac{1}{t+1} + \frac{-4}{2t+1}$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)} dt = \int \frac{1}{t(t+1)(2t+1)} dt$$
$$= \int \left( \frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1} \right) dt$$

$$n \log m = \log m^n$$

$$= \int \left( \frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1} \right) dt$$

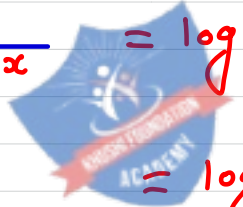
$$= \int \frac{1}{t} dt + \int \frac{1}{t+1} dt - 4 \int \frac{1}{2t+1} dt$$

$$= \log|t| + \log|t+1| - \frac{4 \log|2t+1|}{2} + C$$

$$= \log|t(t+1)| - 2 \log|2t+1| + C$$

$$= \log|t(t+1)| - \log|2t+1|^2 + C$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \log \left| \frac{t(t+1)}{(2t+1)^2} \right| + C$$



$$= \log \left| \frac{e^x(e^x+1)}{(2e^x+1)^2} \right| + C \quad \underline{\text{Ans}}$$

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