Chapter 4: Integration by Partial Fraction (आंशिक भिन्नो द्वारा समाकलन)

Topic(s) : Previous Years Questions (PYQ) Part 4

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UPBTE 2018

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx$$

let $\tan x = t$

$$\frac{d}{dx}\tan x = \frac{dt}{dx}$$

$$\sec^2 x\,dx = dt$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx = \int \frac{1}{(1+t)(2+t)}\,dt$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx = \int \frac{1}{(1+t)(2+t)}\,dt$$

$$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \quad -(1)$$

$$1 = A(2+t) + B(1+t) \quad -(2)$$

put $1+t=0$ in eq. (2) $\quad t=-1$

$1 = A(2-1) + B(0)$

$A = 1$

put $2+t=0$ in eq. (2) $\quad t=-2$

$1 = A(0) + B(1-2)$

$B = -1$

$$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx = \int \frac{1}{(1+t)(2+t)}\,dt = \int \left(\frac{1}{1+t} - \frac{1}{2+t}\right) dt$$

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx = \int \frac{1}{(1+t)(2+t)}\,dt = \int \left(\frac{1}{1+t} - \frac{1}{2+t}\right) dt$$

$$= \int \frac{1}{1+t}\,dt - \int \frac{1}{2+t}\,dt$$

$$= \log|1+t| - \log|2+t| + c$$

$$= \log\left|\frac{1+t}{2+t}\right| + c$$

$$= \log\left|\frac{1+\tan x}{2+\tan x}\right| + c \quad \text{Ans}$$

$$\int \frac{1}{ax+b}\,dx = \frac{\log|ax+b|}{a} + c$$

UPBTE 2017

$$\int \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$$

let $\sin x = t$

$$\frac{d}{dx}\sin x = \frac{dt}{dx}$$

$$\cos x\,dx = dt$$

$$\int \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx = \int \frac{1}{(1+t)(2+t)}\,dt$$

$$= \log\left|\frac{1+t}{2+t}\right| + c$$

$$= \log\left|\frac{1+\sin x}{2+\sin x}\right| + C \quad \text{Ans}$$

UPBTE 2018

$$\int \frac{dx}{x(x^n+1)}$$

$x^{n-1}\,x^1 = x^{n-1+1} = x^n = t$

let $x^n = t$

$\frac{d}{dx}\,x^n = \frac{d}{dx}\,t$

$n\,x^{n-1}\,dx = dt$

$$\int \frac{dx}{x(x^n+1)} = \int \frac{n\,x^{n-1}}{n\,x^{n-1}} \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{1}{t\,(t+1)}\,dt$$

$$\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{1}{t\,(t+1)}\,dt$$

$$\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{1}{t(t+1)}\,dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \quad -①$$

$$1 = A(t+1) + Bt \quad -②$$

put $t=0$ in eq.②

$$1 = A(0+1) + B(0)$$

$$A = 1$$

put $t+1=0$ in eq.②

$t=-1$

$$1 = A(0) + B(-1)$$

$$B = -1$$

$$\frac{1}{t(t+1)} = \frac{1}{t} + \frac{-1}{t+1}$$

$$\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{1}{t(t+1)}\,dt = \frac{1}{n}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{1}{t(t+1)}\,dt$$

$$= \frac{1}{n}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{n}\left[\int \frac{1}{t}\,dt - \int \frac{1}{t+1}\,dt\right]$$

$$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + c$$

$$= \frac{1}{n}\log\left|\frac{t}{t+1}\right| + c$$

$$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$$

Ans

UPBTE 2023

$$\int \frac{dx}{x(x^4+1)}$$

$x^3 x = x^4 = t$

let $x^4 = t$

$\frac{d}{dx} x^4 = \frac{dt}{dx}$

$4x^3 dx = dt$

$$\int \frac{dx}{x(x^4+1)} = \int \frac{4x^3}{4x^3} \frac{dx}{x(x^4+1)} = \frac{1}{4}\int \frac{1}{t(t+1)}\, dt$$

$$= \frac{1}{4} \log \left| \frac{x^4}{x^4+1} \right| + C$$

Q. $\int \frac{dx}{1+3e^x+2e^{2x}}$

$e^{2x} = (e^x)^2$

let $e^x = t$

$\frac{d}{dx} e^x = \frac{d}{dx} t$

$e^x dx = dt$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{e^x\, dx}{e^x(1+3e^x+2(e^x)^2)} = \int \frac{dt}{t(1+3t+2t^2)}$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)}\, dt = \int \frac{1}{t(t+1)(2t+1)}\, dt$$

$2t^2+3t+1 = 2t^2+2t+t+1 = 2t(t+1)+1(t+1)$

$= (t+1)(2t+1)$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)}\,dt = \int \frac{1}{t(t+1)(2t+1)}\,dt$$

$$\frac{1}{t(t+1)(2t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{2t+1} \quad -(1)$$

$$1 = A(t+1)(2t+1) + Bt(2t+1) + Ct(t+1) \quad -(2)$$

$A = 1 \qquad B = 1 \qquad C = -4$

$$\frac{1}{t(t+1)(2t+1)} = \frac{1}{t} + \frac{1}{t+1} + \frac{-4}{2t+1}$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \int \frac{1}{t(2t^2+3t+1)}\,dt = \int \frac{1}{t(t+1)(2t+1)}\,dt$$

$$= \int \left(\frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1}\right) dt$$

$n \log m = \log m^n$

$$= \int \left(\frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1} \right) dt$$

$$= \int \frac{1}{t}\, dt + \int \frac{1}{t+1}\, dt - 4 \int \frac{1}{2t+1}\, dt$$

$$= \log|t| + \log|t+1| - \frac{4 \log|2t+1|}{2} + c$$

$$= \log|t(t+1)| - 2\log|2t+1| + c$$

$$= \log|t(t+1)| - \log|2t+1|^2 + c$$

$$\int \frac{dx}{1+3e^x+2e^{2x}} = \log \left| \frac{t(t+1)}{(2t+1)^2} \right| + c$$

$$= \log \left| \frac{e^x(e^x+1)}{(2e^x+1)^2} \right| + c \quad \underline{\text{Ans}}$$

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