

Chapter 4: Integration by Partial Fraction (आंशिक भिन्नो द्वारा समाकलन)

Topic(s) :

Previous Years Questions (PYQ) Part 3

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Q. $\int \frac{x-1}{(x-3)(x-2)^2} dx$

$$\frac{x-1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \quad \text{--- (1)}$$

$$x-1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3) \quad \text{--- (2)}$$

put $x-3=0$ in eq. (2)
 $x=3$

$$3-1 = A(3-2)^2 + B(0) + C(0)$$

$$2 = A$$

put $x-2=0$ in eq. (2)
 $x=2$

$$2-1 = A(0) + B(0) + C(2-3)$$

$$1 = -C \Rightarrow C = -1$$

$$x-1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

put $x=0$ in eq. (2)

$$0-1 = 2(0-2)^2 + B(0-3)(0-2) + (-1)(0-3)$$

$$-1 = 8 + 6B + 3$$

$$-12 = 6B \Rightarrow B = -2$$

$$\int \frac{x-1}{(x-3)(x-2)^2} dx = \frac{x-1}{(x-3)(x-2)^2} = \frac{2}{(x-3)} + \frac{-2}{(x-2)} + \frac{-1}{(x-2)^2}$$

$$\int \frac{x-1}{(x-3)(x-2)^2} dx = \frac{x-1}{(x-3)(x-2)^2} = \frac{2}{(x-3)} + \frac{-2}{(x-2)} + \frac{-1}{(x-2)^2}$$

$$\int \frac{x-1}{(x-3)(x-2)^2} dx = \int \left(\frac{2}{x-3} - \frac{2}{x-2} - \frac{1}{(x-2)^2} \right) dx$$

$$= 2 \int \frac{1}{x-3} dx - 2 \int \frac{1}{x-2} dx - \int (x-2)^{-2} dx$$

$$= 2 \log|x-3| - 2 \log|x-2| - \frac{(x-2)^{-2+1}}{(-2+1)} + C$$

$$= 2 [\log|x-3| - \log|x-2|] + \frac{1}{x-2} + C$$

$$= 2 \log \left| \frac{x-3}{x-2} \right| + \frac{1}{x-2} + C \quad \underline{\text{Ans}}$$

$$\int \frac{1}{ax+b} dx = \frac{\log|ax+b|}{a} + C$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$$

Q.

$$\int \frac{dx}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2+1} \quad \text{--- (1)}$$

$$1 = A(x^2+1) + (Bx+C)(x+1) \quad \text{--- (2)}$$

put $x+1=0$ in eq. (2)
 $x = -1$

$$1 = A[(-1)^2+1] + (Bx+C)(0)$$

$$1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$$

put $x=0$ in eq. (2)

$$1 = A(x^2+1) + (Bx+C)(x+1) \quad \text{--- (2)}$$

$$1 = \frac{1}{2}(0^2+1) + \frac{(B \cdot 0 + C)}{2}(0+1)$$

$$1 = \frac{1}{2} + C \Rightarrow \boxed{C = \frac{1}{2}}$$

put $x=1$ in eq. (2)

$$1 = A(x^2+1) + (Bx+C)(x+1) \quad \text{--- (2)}$$

$$1 = \frac{1}{2}[1^2+1] + [B(1) + \frac{1}{2}][1+1]$$

$$1 = 1 + 2B + 1 \Rightarrow 2B = -1 \Rightarrow B = -\frac{1}{2}$$

$$\int \frac{dx}{(x+1)(x^2+1)} = \frac{1}{(x+1)} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} \quad \text{--- (1)}$$

$$\int \frac{dx}{(x+1)(x^2+1)} \quad \frac{1}{(x+1)(x^2+1)} = \frac{1/2}{(x+1)} + \frac{-1/2x + 1/2}{x^2+1} \quad \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$$

$$\int \frac{dx}{(x+1)(x^2+1)} = \int \left(\frac{1/2}{(x+1)} + \frac{-1/2x + 1/2}{x^2+1} \right) dx$$

$$= \frac{1}{2} \int \frac{1}{x+1} - \frac{1}{2} \int \frac{x-1}{x^2+1} dx$$

$$= \frac{1}{2} \log |x+1| - \frac{1}{2} \int \frac{x-1}{x^2+1} dx$$

$$= \frac{1}{2} \log |x+1| - \frac{1}{4} \left[\int \frac{2x}{x^2+1} dx - 2 \int \frac{1}{x^2+1} dx \right]$$

$$= \frac{1}{2} \log |x+1| - \frac{1}{4} \left[\log |x^2+1| - 2 \tan^{-1} x \right] + c$$

Ans

$$\int \frac{1}{ax+b} dx = \frac{\log |ax+b|}{a} + c$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

Q.

$$\int \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$$



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Solution to previous practice Problem

$$\int \frac{dx}{x^2 - 4} = \int \frac{dx}{x^2 - 2^2} = \frac{1}{2(2)} \log \left| \frac{x-2}{x+2} \right| + c = \frac{1}{4} \log \left| \frac{x-2}{x+2} \right| + c \quad \underline{\text{Ans}}$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

Practice Problem:

$$\int \frac{x^4}{x^2 + 1} dx$$

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