

Chapter 4: Integration by Partial Fraction ( आंशिक भिन्नो द्वारा समाकलन )

Topic(s) :

Previous Years Questions ( PYQ ) Part 2

Khushi Foundation  
Academy

For PDF Download : Join Telegram Group [KHUSHI FOUNDATION ACADEMY](#)

For any error: [Read pin comment](#)

Q.

$$\int \frac{1}{x^2 - a^2} dx$$

$$\int \frac{1}{ax^2 + bx + c} dx$$

Solution:

$$\frac{1}{x^2 - a^2} = \frac{1}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a} \quad \text{--- (1)}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$1 = A(x-a) + B(x+a) \quad \text{--- (2)}$$

put  $(x+a) = 0$  in eq. (2)  $1 = A(-a-a) + B(0)$   
 $x = -a$

$$A = -\frac{1}{2a}$$

put  $(x-a) = 0$  in eq. (2)  $1 = A(x-a) + B(x+a)$   
 $x = a$   $1 = A(0) + B(a+a)$

$$B = \frac{1}{2a}$$

$$A = -\frac{1}{2a}, B = \frac{1}{2a}$$

A और B का मान समी. (1) में रखने पर

$$\frac{1}{x^2 - a^2} = \frac{1}{(x+a)(x-a)} = \frac{-\frac{1}{2a}}{x+a} + \frac{\frac{1}{2a}}{x-a}$$

$$\frac{1}{x^2 - a^2} = \frac{1}{(x+a)(x-a)} = \frac{-\frac{1}{2}a}{(x+a)} + \frac{\frac{1}{2}a}{(x-a)}$$

$$\int \frac{1}{x^2 - a^2} dx = \int \left( \frac{-\frac{1}{2}a}{(x+a)} + \frac{\frac{1}{2}a}{(x-a)} \right) dx$$

$$\int \frac{1}{bx+a} dx = \frac{\log|bx+a|}{b} + C$$

$$= -\frac{1}{2a} \int \frac{1}{(x+a)} dx + \frac{1}{2a} \int \frac{1}{(x-a)} dx$$

$$= -\frac{1}{2a} \log|x+a| + \frac{1}{2a} \log|x-a| + C$$

$$= \frac{1}{2a} \left( -\log|x+a| + \log|x-a| \right) + C$$

$$= \frac{1}{2a} \left( \log|x-a| - \log|x+a| \right) + C$$

$$\log m - \log n = \log \frac{m}{n}$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

Ans

Q.  $\int \frac{dx}{x^2 - 4} = \int \frac{dx}{x^2 - \underset{(a)}{2^2}} = \frac{1}{2(2)} \log \left| \frac{x-2}{x+2} \right| + c = \frac{1}{4} \log \left| \frac{x-2}{x+2} \right| + c$  Ans

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$



Khushi Foundation  
Academy

Q.  $\int \frac{1}{a^2 - x^2} dx$       $\int \frac{1}{ax^2 + bx + c} dx$

$$\frac{1}{a^2 - x^2} = \frac{1}{(a+x)(a-x)} = \frac{A}{(a+x)} + \frac{B}{(a-x)} \quad \text{--- (1)}$$

$$1 = A(a-x) + B(a+x) \quad \text{--- (2)}$$

put  $(a+x) = 0$  in eq. (2)  $x = -a$

$$A = \frac{1}{2a}$$

put  $(a-x) = 0$  in eq. (2)  $x = a$

$$B = \frac{1}{2a}$$

A और B का मान समी. (1) में रखने पर

$$\frac{1}{a^2 - x^2} = \frac{1}{(a+x)(a-x)} = \frac{1/2a}{(a+x)} + \frac{1/2a}{(a-x)}$$

$$\frac{1}{a^2 - x^2} = \frac{1}{(a+x)(a-x)} = \frac{1/2a}{(a+x)} + \frac{1/2a}{(a-x)}$$

$$\int \frac{1}{px+q} dx = \frac{\log|px+q|}{p} + c$$

$$\int \frac{1}{a^2 - x^2} dx = \int \left( \frac{1/2a}{(a+x)} + \frac{1/2a}{(a-x)} \right) dx$$

$$= \frac{1}{2a} \int \frac{1}{(a+x)} dx + \frac{1}{2a} \int \frac{1}{(a-x)} dx$$

$$= \frac{1}{2a} \log|a+x| + \frac{1}{2a} \log|a-x| + C$$

$$= \frac{1}{2a} \log|a+x| - \frac{1}{2a} \log|a-x| + C$$

$$= \frac{1}{2a} \left( \log|a+x| - \log|a-x| \right) + C$$

$$\log m - \log n = \log \frac{m}{n}$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

Q.  $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

Q.  $\int \frac{dx}{4 - x^2} = \int \frac{dx}{2^2 - x^2} = \frac{1}{2(2)} \log \left| \frac{2+x}{2-x} \right| + c = \frac{1}{4} \log \left| \frac{2+x}{2-x} \right| + c$

$\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$



$\int \frac{1}{a^2 - x^2} dx$

Q.  $\int \frac{dx}{4x^2 - 9} = \int \frac{dx}{4(x^2 - \frac{9}{4})}$        $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

$$= \frac{1}{4} \int \frac{dx}{x^2 - \left(\frac{3}{2}\right)^2} = \frac{1}{4} \times \frac{1}{2 \times \frac{3}{2}} \log \left| \frac{\left(\frac{x}{1} - \frac{3}{2}\right)}{\left(\frac{x}{1} + \frac{3}{2}\right)} \right| + C$$

$$= \frac{1}{12} \log \left| \frac{2x-3}{2x+3} \right| + C$$


---

Q.  $\int \frac{dx}{25 - 9x^2} = \int \frac{dx}{9\left(\frac{25}{9} - x^2\right)}$        $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

$$= \frac{1}{9} \int \frac{dx}{\left(\frac{5}{3}\right)^2 - x^2} = \frac{1}{9} \times \frac{1}{2 \times \frac{5}{3}} \log \left| \frac{\left(\frac{5}{3} + \frac{x}{1}\right)}{\left(\frac{5}{3} - \frac{x}{1}\right)} \right| + C$$

$$= \frac{1}{30} \log \left| \frac{5+3x}{5-3x} \right| + C \quad \underline{\text{Ans}}$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

Q.  $\int \frac{dx}{x^2 + 4} = \int \frac{dx}{x^2 + 2^2} = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$  Ans

Q.  $\int \frac{dx}{4 + 9x^2} = \int \frac{dx}{9\left[\frac{4}{9} + x^2\right]} = \frac{1}{9} \int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2} = \frac{1}{9} \times \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{x}{2/3}\right) + C$   
 $= \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$

Ans

Summary:

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

---



Khushi Foundation  
Academy

Q.

$$\int \frac{x^2 + 1}{x(x^2 - 1)} dx$$

$$\frac{f(x)}{(x - \alpha)(ax^2 + bx + c)}$$

$$\int \frac{x^2 + 1}{x(x^2 - 1^2)} dx = \int \frac{x^2 + 1}{x(x+1)(x-1)} dx$$



Khushi Foundation  
Academy

Solution:

$$\frac{x^2 + 1}{x(x^2 - 1^2)} = \frac{x^2 + 1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \quad \text{--- (1)}$$

$$x^2 + 1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1) \quad \text{--- (2)}$$

put  $x=0$  in eq. (2)  $0^2 + 1 = A(0+1)(0-1) + B \cdot 0(0-1) + C \cdot 0(0+1)$

$$1 = A(0+1)(0-1) + 0 + 0$$

$$A = -1$$

put  $(x+1)=0$  in eq. (2)  $x^2 + 1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$   
 $x = -1$   $(-1)^2 + 1 = A(0) + B(-1)(-1-1) + C(0)$

$$2 = 2B$$

$$B = 1$$

put  $(x-1)=0$  in eq. (2)  $x^2 + 1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$   
 $x = 1$   $(1)^2 + 1 = A(0) + B(0) + C(1)(1+1)$

$$2 = 2C$$

$$C = 1$$

$$A = -1, B = 1, C = 1$$

$$\frac{x^2+1}{x(x^2-1^2)} = \frac{x^2+1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \quad \text{--- } \textcircled{1} \quad A=-1, B=1, C=1$$

$$\int \frac{x^2+1}{x(x^2-1)} dx = \int \left( \frac{-1}{x} + \frac{1}{(x+1)} + \frac{1}{(x-1)} \right) dx$$

$$= \int \frac{-1}{x} dx + \int \frac{1}{(x+1)} dx + \int \frac{1}{(x-1)} dx$$

$$= -\log|x| + \log|x+1| + \log|x-1| + C$$

$$= -\log|x| + \log|(x+1)(x-1)| + C$$

$$= \log|x^2-1| - \log|x| + C$$

$$\int \frac{x^2+1}{x(x^2-1)} dx = \log \left| \frac{x^2-1}{x} \right| + C$$

ANS

Practice Problem:

$$\int \frac{dx}{6x - x^2 - 5}$$

$$\int \frac{1}{ax^2 + bx + c} dx$$

Solution:

$$\int \frac{dx}{-(-6x + x^2 + 5)} = \int \frac{-dx}{(x^2 - 6x + 5)} = \int \frac{-dx}{x^2 - 5x - x + 5} = \int \frac{-dx}{x(x-5) - 1(x-5)}$$

$$\int \frac{-dx}{(x-5)(x-1)} \quad \frac{-1}{(x-5)(x-1)} = \frac{A}{x-5} + \frac{B}{x-1} \quad \text{--- (1)}$$

put  $(x-5) = 0$  in eq. (2)  
 $x = 5$

$$-1 = A(x-1) + B(x-5) \quad \text{--- (2)}$$

$$-1 = A(5-1) + B(0)$$

$$A = \frac{-1}{4}$$

put  $(x-1) = 0$  in eq. (2)  
 $x = 1$

$$-1 = A(0) + B(1-5)$$

$$B = \frac{1}{4}$$

A और B का मान समी. (1) में रखने पर

$$\int \frac{-dx}{(x-5)(x-1)} = \frac{-1}{(x-5)(x-1)} = \frac{-1/4}{x-5} + \frac{1/4}{x-1} \quad \text{--- (1)}$$

$$\int \frac{-dx}{(x-5)(x-1)} \quad \frac{-1}{(x-5)(x-1)} = \frac{-\frac{1}{4}}{(x-5)} + \frac{\frac{1}{4}}{(x-1)} \quad \text{--- (1)}$$

$$\int \frac{-dx}{(x-5)(x-1)} = \int \left( \frac{-\frac{1}{4}}{(x-5)} + \frac{\frac{1}{4}}{(x-1)} \right) dx$$

$$= -\frac{1}{4} \int \frac{1}{(x-5)} dx + \frac{1}{4} \int \frac{1}{(x-1)} dx$$

$\int \frac{1}{px+q} dx = \frac{\log|px+q|}{p} + c$

$$= -\frac{1}{4} \log|x-5| + \frac{1}{4} \log|x-1| + c$$

$$= \frac{1}{4} \left( -\log|x-5| + \log|x-1| \right) + c$$

$$= \frac{1}{4} \left( \log|x-1| - \log|x-5| \right) + c$$

$$\log m - \log n = \log \frac{m}{n}$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x-5} \right| + c$$

Ans:

Practice Problem

$$\int \frac{dx}{x^2 - 4}$$

COMMENT

Next video

Previous Years Questions (PYQ) Part 3



Khushi Foundation  
Academy

LIKE

SHARE

SUBSCRIBE

YouTube Channel

Khushi Foundation Academy



THANK YOU