

Chapter 4: Integration by Partial Fraction (आंशिक भिन्नो द्वारा समाकलन)

Topic(s) :

Previous Years Questions (PYQ) Part 1

$$\int \frac{1}{ax+b} dx = \frac{\log|ax+b|}{a} + c$$

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Q. $\int \frac{1}{(x+2)(x+3)} dx$

$$\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} \quad \text{--- ①}$$

$$1 = A(x+3) + B(x+2) \quad \text{--- ②};$$

put $x+2=0$ in eq. ②
 $x = -2$

$$1 = A(-2+3) + B(0)$$

$$1 = A$$

$$1 = A(x+3) + B(x+2)$$

put $x+3=0$ in eq. ②
 $x = -3$

$$1 = 1(0) + B(-3+2)$$

$$1 = -B \Rightarrow -1 = B$$

$$\frac{1}{(x+2)(x+3)} = \frac{1}{x+2} + \frac{-1}{x+3} \quad \text{--- ③}$$

$$\int \frac{1}{(x+2)(x+3)} dx$$

$$\int \frac{1}{ax+b} dx = \frac{\log|ax+b|}{a} + c$$

$$\frac{1}{(x+2)(x+3)} = \frac{1}{x+2} + \frac{-1}{x+3} \quad \text{--- (3)}$$

$$\int \frac{1}{(x+2)(x+3)} dx = \int \left(\frac{1}{x+2} - \frac{1}{x+3} \right) dx$$

$$= \int \frac{1}{x+2} dx - \int \frac{1}{x+3} dx$$

$$= \log|x+2| - \log|x+3| + c \quad ; \log m - \log n = \log \frac{m}{n}$$

$$= \log \left| \frac{x+2}{x+3} \right| + c \quad \underline{\text{Ans}}$$

Q. $\int \frac{(x-1)}{(x-2)(x+3)} dx$

$$\frac{(x-1)}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)} \quad \text{--- ①}$$

$$(x-1) = A(x+3) + B(x-2) \quad \text{--- ②} \quad \times (x-2)(x+3)$$

put $x-2=0$ in eq. ②
 $x=2$

$$1 = 5A \Rightarrow A = \frac{1}{5}$$

$$(x-1) = A(x+3) + B(x-2) \quad \text{--- ②}$$

put $x+3=0$ in eq. ②
 $x=-3$

$$-4 = -5B \Rightarrow B = \frac{4}{5}$$

$$\frac{(x-1)}{(x-2)(x+3)} = \frac{(1/5)}{(x-2)} + \frac{(4/5)}{(x+3)} \quad \text{--- ③}$$

$$\frac{(x-1)}{(x-2)(x+3)} = \frac{(1/5)}{(x-2)} + \frac{(4/5)}{(x+3)} \quad \text{--- (3)}$$

$$\begin{aligned} \int \frac{(x-1)}{(x-2)(x+3)} dx &= \int \left[\frac{(1/5)}{(x-2)} + \frac{(4/5)}{x+3} \right] dx \\ &= \frac{1}{5} \int \frac{1}{x-2} dx + \frac{4}{5} \int \frac{1}{x+3} dx \\ &= \frac{1}{5} \log |x-2| + \frac{4}{5} \log |x+3| + C \end{aligned}$$

Ans



Q.

$$\int \frac{dx}{2x^2 + x - 1} = \int \frac{dx}{(x+1)(2x-1)}$$

$$\int \frac{dx}{ax^2 + bx + c}$$

$$\begin{aligned} & 2x^2 + x - 1 \\ & 2x^2 + 2x - x - 1 \\ & 2x(x+1) - 1(x+1) \\ & (x+1)(2x-1) \end{aligned}$$

$$\frac{1}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1} \quad \text{--- (1)}$$

$$1 = A(2x-1) + B(x+1) \quad \text{--- (2)}$$

put $x+1=0$ in eq. (2)
 $x = -1$

$$1 = A(2(-1) - 1) + B(0)$$

$$1 = -3A$$

$$A = -\frac{1}{3}$$

put $2x-1=0$ in eq. (2)

$$2x = 1$$

$$x = \frac{1}{2}$$

$$1 = A(2x-1) + B(x+1) \quad \text{--- (2)}$$

$$1 = -\frac{1}{3}(0) + B\left(\frac{1}{2} + 1\right) \Rightarrow 1 = B \frac{3}{2} \Rightarrow B = \frac{2}{3}$$

$$\frac{1}{(x+1)(2x-1)} = \frac{(-1/3)}{x+1} + \frac{2/3}{2x-1} \quad \text{--- (3)}$$

$$\frac{1}{(x+1)(2x-1)} = \frac{(-1/3)}{x+1} + \frac{2/3}{2x-1} \quad - \textcircled{3}$$

$$\int \frac{dx}{2x^2+x-1} = \int \frac{dx}{(x+1)(2x-1)} = \int \left(\frac{-1/3}{x+1} + \frac{2/3}{2x-1} \right) dx$$

$$= -\frac{1}{3} \int \frac{1}{x+1} dx + \frac{2}{3} \int \frac{1}{2x-1} dx$$

$$= -\frac{1}{3} \log|x+1| + \frac{2}{3} \frac{\log|2x-1|}{2} + C$$

$$= \frac{1}{3} \log|2x-1| - \frac{1}{3} \log|x+1| + C$$

$$= \frac{1}{3} \left[\log|2x-1| - \log|x+1| \right] + C$$

$$= \frac{1}{3} \log \left| \frac{2x-1}{x+1} \right| + C \quad \text{Ans}$$



Q.

$$\int \frac{x^2+1}{x(x^2-1)} dx$$

Solution:

$$\int \frac{x^2+1}{x(x^2-1^2)} dx$$

$$\int \frac{x^2+1}{x(x+1)(x-1)} dx$$

$$\frac{x^2+1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \quad \text{--- (1)}$$

$$x^2+1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1) \quad \text{--- (2)}$$

$$A = -1$$

$$B = 1$$

$$C = 1$$

$$\text{Ans: } \log \left| \frac{x^2-1}{x} \right| + C$$

$$\log|x+1| + \log|x-1|$$

Q. $\int \frac{x+3}{x-2} dx$

$$\int \frac{x-2+2+3}{x-2} dx = \int \left(\frac{x-2+5}{x-2} \right) dx$$

$$= \int \left(\frac{x-2}{x-2} + \frac{5}{x-2} \right) dx$$

$$= \int 1 dx + 5 \int \frac{1}{x-2} dx$$

$$= x + 5 \log |x-2| + C \quad \underline{\text{Ans}}$$



Summary:

1 $\frac{f(x)}{(x-\alpha)(x-\beta)(x-\gamma)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\gamma)}$ Non-repeated linear factors
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Practice Problem:

$$\int \frac{dx}{6x - x^2 - 5}$$

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