Topic(s): Previous Years Questions (PYQ) & Practice Problems

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UP BTE 2023,....

Q.1: $\int \left(x - \frac{1}{x}\right)^2 dx$

Solution:

$$\int \left(x - \frac{1}{x}\right)^2 dx$$

$$= \int \left[x^2 + \left(\frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x}\right] dx$$

$$= \int \left(x^2 + \frac{1}{x^2} - 2\right) dx$$

$$= \int x^2 dx + \int x^{-2} dx - 2\int dx$$

$$= \frac{x^3}{3} + \frac{x^{-2+1}}{-2+1} - 2x + c$$

$$= \frac{x^3}{3} - \frac{1}{x} - 2x + c \quad \text{Ans}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\int x^n dx = \frac{x^{n+1}}{(n+1)} + c \quad ; n \neq -1$$

UP BTE 2017,....

Q.2: $\int \left(1+\frac{1}{x}\right)^2 dx$

Solution:

$$\int \left(1+\frac{1}{x}\right)^2 dx = \int \left[1^2 + \left(\frac{1}{x}\right)^2 + 2(1)\frac{1}{x}\right] dx$$

; $(a+b)^2 = a^2 + b^2 + 2ab$

$$= \int \left(1 + \frac{1}{x^2} + \frac{2}{x}\right) dx$$

$$= \int 1\, dx + \int \frac{1}{x^2} dx + \int \frac{2}{x}\, dx$$

$$= 1\int dx + \int x^{-2} dx + 2\int \frac{1}{x} dx$$

; $\int x^n dx = \frac{x^{n+1}}{(n+1)} + c$

$$= x + \frac{x^{-2+1}}{-2+1} + 2\log|x| + c$$

$$= x + \frac{x^{-1}}{-1} + 2\log|x| + c$$

; $\int \frac{1}{x} dx = \log|x| + c$

$$= x - \frac{1}{x} + 2\log|x| + c$$

Ans

Q.3: $\int \frac{x^2}{x^2+1}\,dx$

Solution:

$$\int \frac{x^2+1-1}{x^2+1}\,dx \qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right) dx$$

$$\int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$= 1\int dx - \int \frac{1}{x^2+1}\,dx$$

$$= x - \tan^{-1}x + c$$

Ans

Review:

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

; $\sin^2 x + \cos^2 x = 1$

UP BTE 2016,....

Q.4: $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$

Solution:

$$\int \frac{1-\cos 2x}{1+\cos 2x}\,dx = \int \frac{2\sin^2 x}{2\cos^2 x}\,dx$$

$$= \int \tan^2 x\,dx$$

$$= \int (\sec^2 x - 1)\,dx$$

$$= \int \sec^2 x\,dx - 1\int dx$$

$$= \tan x - x + c \quad \text{Ans}$$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

Q.5: $\int \frac{dx}{1+\sin x}$

Solution:

$$\int \frac{1}{(1+\sin x)} \frac{(1-\sin x)}{(1-\sin x)}\, dx$$

$$\int \frac{1-\sin x}{1^2-\sin^2 x}\, dx$$

$$\int \frac{1-\sin x}{\cos^2 x}\, dx$$

$$\int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$$

$$\int \left(\sec^2 x - \frac{\sin x}{\cos x \cos x}\right) dx = \int \sec^2 x\, dx - \int \sec x \tan x\, dx$$

$$= \tan x - \sec x + C \quad \text{Ans}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

Note 1:

$$\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx = \int \sec x \tan x\,dx = \sec x + c$$

Note 2:

$$\int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\,dx = \int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$$

UP BTE 1993,....

Q.6 $\int \frac{4 - 5\sin x}{\cos^2 x}\, dx$

Solution:

$$\int \frac{4 - 5\sin x}{\cos^2 x}\, dx$$

$$= \int \left(\frac{4}{\cos^2 x} - \frac{5\sin x}{\cos x \cos x} \right) dx$$

$$= 4\int \sec^2 x\, dx - 5\int \sec x \tan x\, dx$$

$$= 4\tan x - 5\sec x + c \quad \text{Ans}$$

UP BTE 1998,....

Q.7: $\int \frac{1+\sin x}{\cos^2 x}\, dx$

Solution:

$$\int \frac{1+\sin x}{\cos^2 x}\, dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos x \cos x} \right) dx$$

$$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$$

$$= \tan x + \sec x + C \quad \text{Ans}$$

Q.8: $\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx$

Solution:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}\, dx$$

$$= \int \left(\frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int (\operatorname{cosec}^2 x - \sec^2 x)\, dx$$

$$= \int \operatorname{cosec}^2 x\, dx - \int \sec^2 x\, dx$$

$$= -\cot x - \tan x + c$$

$$= -(\cot x + \tan x) + c$$ Ans

UP BTE 1999,....

Q.9: $\int \frac{\sec x + \tan x}{\sec x - \tan x}\, dx$

Solution:

$$\int \frac{(\sec x + \tan x)}{(\sec x - \tan x)} \frac{(\sec x + \tan x)}{(\sec x + \tan x)}\, dx$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

$$\int \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x}\, dx$$

$$\int \left(\frac{\sec^2 x + \tan^2 x + 2 \sec x \tan x}{1} \right) dx$$

$$\int (\sec^2 x + \sec^2 x - 1 + 2 \sec x \tan x)\, dx$$

$$\int (2 \sec^2 x - 1 + 2 \sec x \tan x)\, dx$$

$$2 \int \sec^2 x\, dx - 1 \int dx + 2 \int \sec x \tan x\, dx$$

$$2 \tan x - x + 2 \sec x + c \quad \text{Ans.}$$

UP BTE 1999,....

Q.9: $\int \frac{\sec x + \tan x}{\sec x - \tan x}\, dx$

Solution:

$$\int \frac{\sec x + \tan x}{\sec x - \tan x}\, dx$$

$$\int \frac{\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)\cos x}{\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\cos x}\, dx$$

$$\int \frac{1 + \sin x}{1 - \sin x}\, dx$$

$$\int \frac{(1+\sin x)}{(1-\sin x)} \frac{(1+\sin x)}{(1+\sin x)}\, dx$$

$$\int \frac{(1+\sin x)^2}{1^2 - \sin^2 x}$$

$$= \int \frac{1^2 + \sin^2 x + 2(1)\sin x}{1 - \sin^2 x}\, dx$$

$$= \int \frac{1 + \sin^2 x + 2\sin x}{\cos^2 x}\, dx$$

$$\int \left(\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} + 2\frac{\sin x}{\cos^2 x}\right) dx$$

$$\int \left(\sec^2 x + \tan^2 x + \frac{2\sin x}{\cos x \cos x}\right) dx$$

$$\int (\sec^2 x + \sec^2 x - 1 + 2\sec x \tan x)\, dx$$

$$\int (2\sec^2 x - 1 + 2\sec x \tan x)\, dx$$

$$2\int \sec^2 x\, dx - 1\int dx + 2\int \sec x \tan x\, dx$$

$$2\tan x - x + 2\sec x + C \quad \text{Ans}$$

Q.10: $\int \frac{1}{1+\cos x}\, dx$

Solution:

$$\int \frac{1}{(1+\cos x)} \frac{(1-\cos x)}{(1-\cos x)}\, dx$$

$$\int \frac{1-\cos x}{1-\cos^2 x}\, dx$$

$$\int \frac{1-\cos x}{\sin^2 x}\, dx$$

$$\int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin x \sin x} \right) dx$$

$$= \int \operatorname{cosec}^2 x\, dx - \int \operatorname{cosec} x \cot x\, dx$$

$$= -\cot x - (-\operatorname{cosec} x) + c = -\cot x + \operatorname{cosec} x + c$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

HW Question:

$$\int \frac{1}{1-\cos x}\, dx$$

COMMENT

Next video: Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)

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