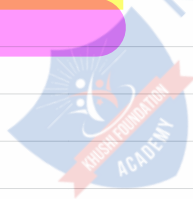


Topic(s):

Previous Years Questions (PYQ) & Practice Problems

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Q.1: $\int \left(x - \frac{1}{x}\right)^2 dx$

Solution:

$$\int \left(x - \frac{1}{x}\right)^2 dx$$

$$= \int \left[x^2 + \left(\frac{1}{x}\right)^2 - 2x \cdot \frac{1}{x} \right] dx$$

$$= \int \left(x^2 + \frac{1}{x^2} - 2 \right) dx$$

$$= \int x^2 dx + \int x^{-2} dx - 2 \int dx$$

$$= \frac{x^3}{3} + \frac{x^{-2+1}}{-2+1} - 2x + C$$

$$= \frac{x^3}{3} - \frac{1}{x} - 2x + C \quad \underline{\text{Ans}}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\int x^n dx = \frac{x^{n+1}}{(n+1)} + C ; n \neq -1$$

Q.2: $\int \left(1 + \frac{1}{x}\right)^2 dx$

Solution:

$$\int \left(1 + \frac{1}{x}\right)^2 dx = \int \left[1^2 + \left(\frac{1}{x}\right)^2 + 2(1) \frac{1}{x}\right] dx \quad ; (a+b)^2 = a^2 + b^2 + 2ab$$

$$= \int \left(1 + \frac{1}{x^2} + \frac{2}{x}\right) dx$$

$$= \int 1 dx + \int \frac{1}{x^2} dx + \int \frac{2}{x} dx$$

$$= \int dx + \int x^{-2} dx + 2 \int \frac{1}{x} dx$$

$$= x + \frac{x^{-2+1}}{-2+1} + 2 \log|x| + c$$

$$= x + \frac{x^{-1}}{-1} + 2 \log|x| + c$$

$$= x - \frac{1}{x} + 2 \log|x| + c$$

Ans

$$; \int x^n dx = \frac{x^{n+1}}{(n+1)} + c$$

$$; \int \frac{1}{x} dx = \log|x| + c$$

Q.3: $\int \frac{x^2}{x^2+1} dx$

Solution:

$$\int \frac{x^2+1-1}{x^2+1} dx$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} \right) dx$$

$$\int \left(1 - \frac{1}{x^2+1} \right) dx$$

$$= \int dx - \int \frac{1}{x^2+1} dx$$

$$= x - \tan^{-1} x + C$$

Ans

Review:

$$\cos 2p = 1 - 2 \sin^2 p$$

$$2 \sin^2 p = 1 - \cos 2p$$

$$\sin^2 p = \frac{1 - \cos 2p}{2}$$

$$\cos 2p = \cos^2 p - \sin^2 p$$

$$\cos 2p = \frac{1 - \tan^2 p}{1 + \tan^2 p}$$

$$\cos 2p = 2 \cos^2 p - 1$$

$$\cos 2p + 1 = 2 \cos^2 p$$

$$\frac{\cos 2p + 1}{2} = \cos^2 p$$

$$\cos^2 p = \frac{1 + \cos 2p}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

UP BTE 2016,....

Q.4: $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$

Solution:

$$\int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \int \frac{2 \sin^2 x}{2 \cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1) dx$$

$$= \int \sec^2 x dx - \int dx$$

$$= \tan x - x + c \quad \underline{\text{Ans}}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$



Q.5: $\int \frac{dx}{1 + \sin x}$

Solution:

$$\int \frac{1}{(1 + \sin x)(1 - \sin x)} dx$$

$$\int \frac{1 - \sin x}{1^2 - \sin^2 x} dx$$

$$\int \frac{1 - \sin x}{\cos^2 x} dx$$

$$\int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx$$

$$\int \left(\sec^2 x - \frac{\sin x}{\cos x \cos x} \right) dx = \int \sec^2 x dx - \int \sec x \tan x dx$$
$$= \tan x - \sec x + C \text{ Ans}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

Note 1:

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{\cos x \cos x} dx = \int \sec x \tan x dx = \sec x + c$$

Note 2:

$$\int \frac{\cos x}{\sin^2 x} dx = \int \frac{\cos x}{\sin x \sin x} dx = \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$$



UP BTE 1993,....

Q.6 $\int \frac{4 - 5 \sin x}{\cos^2 x} dx$

Solution:

$$\begin{aligned} & \int \frac{4 - 5 \sin x}{\cos^2 x} dx \\ &= \int \left(\frac{4}{\cos^2 x} - \frac{5 \sin x}{\cos x \cos x} \right) dx \\ &= 4 \int \sec^2 x dx - 5 \int \sec x \tan x dx \\ &= 4 \tan x - 5 \sec x + c \end{aligned}$$

Ans



UP BTE 1998,....

Q.7: $\int \frac{1 + \sin x}{\cos^2 x} dx$

Solution:

$$\begin{aligned} & \int \frac{1 + \sin x}{\cos^2 x} dx \\ &= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos x \cos x} \right) dx \\ &= \int \sec^2 x dx + \int \sec x \tan x dx \\ &= \tan x + \sec x + C \quad \underline{\text{Ans}} \end{aligned}$$



Q.8: $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

Solution:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \left(\frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int (\operatorname{cosec}^2 x - \sec^2 x) dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\cot x - \tan x + C$$

$$= -(\cot x + \tan x) + C \quad \underline{\text{Ans}}$$

Q.9: $\int \frac{\sec x + \tan x}{\sec x - \tan x} dx$

Solution:

$$\int \frac{(\sec x + \tan x)(\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)} dx$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

$$\int \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x} dx$$

$$\int \left(\frac{\sec^2 x + \tan^2 x + 2 \sec x \tan x}{1} \right) dx$$

$$\int (\sec^2 x + \sec^2 x - 1 + 2 \sec x \tan x) dx$$

$$\int (2 \sec^2 x - 1 + 2 \sec x \tan x) dx$$

$$2 \int \sec^2 x dx - 1 \int dx + 2 \int \sec x \tan x dx$$

$$2 \tan x - x + 2 \sec x + c \quad \text{Ans.}$$

Q.9:

$$\int \frac{\sec x + \tan x}{\sec x - \tan x} dx$$

Solution:

$$\int \frac{\sec x + \tan x}{\sec x - \tan x} dx$$

$$\int \frac{\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) \cos x}{\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) \cos x} dx$$

$$\int \frac{1 + \sin x}{1 - \sin x} dx$$

$$\int \frac{(1 + \sin x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} dx$$

$$\int \frac{(1 + \sin x)^2}{1^2 - \sin^2 x}$$

$$= \int \frac{1^2 + \sin^2 x + 2(1)\sin x}{1 - \sin^2 x} dx$$

$$= \int \frac{1 + \sin^2 x + 2 \sin x}{\cos^2 x} dx$$

$$\int \left(\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} + \frac{2 \sin x}{\cos^2 x} \right) dx$$

$$\int \left(\sec^2 x + \tan^2 x + \frac{2 \sin x}{\cos x \cos x} \right) dx$$

$$\int (\sec^2 x + \sec^2 x - 1 + 2 \sec x \tan x) dx$$

$$\int (2 \sec^2 x - 1 + 2 \sec x \tan x) dx$$

$$2 \int \sec^2 x dx - \int dx + 2 \int \sec x \tan x dx$$

$$2 \tan x - x + 2 \sec x + C \quad \text{Ans}$$

Q.10: $\int \frac{1}{1 + \cos x} dx$

Solution:

$$\int \frac{1}{(1 + \cos x)(1 - \cos x)} dx$$

$$\int \frac{1 - \cos x}{1 - \cos^2 x} dx$$

$$\int \frac{1 - \cos x}{\sin^2 x} dx$$

$$\int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin x \sin x} \right) dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \operatorname{cosec} x \cot x dx$$

$$= -\cot x - (-\operatorname{cosec} x) + c = -\cot x + \operatorname{cosec} x + c$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

HW Question:

$$\int \frac{1}{1 - \cos x} dx$$

COMMENT

Next video:

Chapter 2: Integration by Substitution (प्रतिस्थापन द्वारा समाकलन)



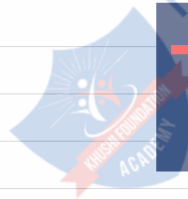
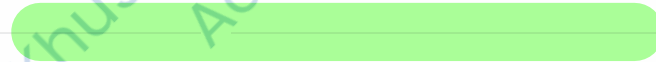
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