Topic : Problems based on below formulae

(1) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty$ terms

(2) $T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$

(3) Coefficient of x^r

(4) ${}^nC_r = {}^nC_{n-r}$ | ${}^nC_x = {}^nC_y$; $x \neq y$

Then $x + y = n$

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Q.1: $(3+2x)^{3/2}$ का चार पदों तक विस्तार करें।

Expand (3+2x)^(3/2) upto 4 terms

Solution:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots \infty \text{ terms}$$

$$(3+2x)^{3/2} = \left\{3\left(1+\frac{2}{3}x\right)\right\}^{\frac{3}{2}}$$

$$= 3^{3/2}\left[1+\frac{2}{3}x\right]^{3/2}$$

$$= 3^{3/2}\left[1 + \frac{3}{2}\times\frac{2}{3}x + \frac{1}{2\times1}\,\frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{2}{3}x\right)^2 + \frac{1}{3\times2\times1}\,\frac{3}{2}\times\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)\left(\frac{2}{3}x\right)^3 + \dots\right]$$

$$= 3^{3/2}\left[1 + x + \frac{1}{2}\times\frac{3}{2}\times\frac{1}{2}\times\frac{2^2x^2}{3^2} + \frac{1}{3\times2}\times\frac{3}{2}\times\frac{1}{2}\left(-\frac{1}{2}\right)\frac{2^3}{3^3}x^3 + \dots\right]$$

$$3^{3/2} = \left(3^3\right)^{1/2} = \sqrt{3^3} = \sqrt{3\times3\times3} = 3\sqrt{3}$$

$$= 3^{3/2}\left[1 + x + \frac{1}{2}\times\frac{3}{2}\times\frac{1}{2}\times\frac{2^2 x^2}{3^2} + \frac{1}{3\times2}\times\frac{3}{2}\times\frac{1}{2}\left(-\frac{1}{2}\right)\frac{2^3}{3^3}x^3 + \dots\right]$$

$$= 3\sqrt{3}\left[1 + x + \frac{1}{6}x^2 - \frac{1}{54}x^3 + \dots\right]$$

Ans

Q.2 : $(1-x^2)^{7/2}$ का चार पदों तक विस्तार करें ।

Expand (1-x^2)^(7/2) upto 4 terms

Solution :

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$$(1-x^2)^{7/2} = \{1+(-x^2)\}^{7/2}$$

$$= 1 + \frac{7}{2}(-x^2) + \frac{1}{2!}\frac{7}{2}\left(\frac{7}{2}-1\right)(-x^2)^2 + \frac{1}{3!}\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right)(-x^2)^3 + \cdots$$

$$= 1 - \frac{7}{2}x^2 + \frac{1}{2\times 1}\frac{7}{2}\times\frac{5}{2}x^4 + \frac{1}{3\times 2\times 1}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}(-x^6) + \cdots$$

$$(1-x^2)^{7/2} = 1 - \frac{7}{2}x^2 + \frac{35}{8}x^4 - \frac{35}{16}x^6 + \cdots$$ Ans

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

a   b

3   4   5   6   7   8   9   10

$b - a + 1$

$10 - 3 + 1$

8

a   b

-10 -9 -8 -7 -6 -5 -4 -3

$b - a + 1$

$-3-(-10)+1$

$-3+10+1$

$-3+11$

8

Q.3: $(1+x^2)^{-3}$ के विस्तार में व्यापक पद बतायें।

Find general term in the expansion of (1+x^2)^(-3)

Solution:

$(1+x)^n$: $T_{r+1} = \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \dfrac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$

$$T_{r+1} = \frac{-3(-3-1)(-3-2)\cdots(-3-r+1)}{r!}(x^2)^r$$

$$T_{r+1} = \frac{(-3)(-4)(-5)(-6)\cdots(-2-r)}{r!}x^{2r}$$

$-3-(-2-r)+1$

$-3+2+r+1$

r

$-4 = (-1)4$

$$T_{r+1} = \frac{(-3)(-4)(-5)(-6)\cdots(-2-r)}{r!}x^{2r}$$

$$T_{r+1} = \frac{(-1)^r(3)(4)(5)(6)\cdots(r+2)}{r!}x^{2r}$$

$$T_{r+1} = (-1)^r\frac{1\times2\times(3)(4)(5)(6)\cdots r(r+1)(r+2)}{1\times2\; r!}x^{2r}$$

$$T_{r+1} = (-1)^r\frac{r!\,(r+1)(r+2)}{2\,r!}x^{2r} \quad ;[1\times2\times3\times4 = 4!]$$

$$T_{r+1} = (-1)^r\frac{(r+1)(r+2)}{2}x^{2r}$$ Ans

Correction credit: Ashutosh Chauhan (CSE GP MAU)

Q.4: $(1-x^2)^{-3}$ के विस्तार में व्यापक पद बतायें।

Find general term in the expansion of (1- x^2)^(-3)

Solution:

$(1+x)^n$. $\quad T_{r+1} = \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \dfrac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$

$(1-x^2)^{-3} = (1+(-x^2))^{-3}$; यहाँ (here) : $n=-3$ $\quad x \longrightarrow (-x^2)$

$T_{r+1} = \dfrac{-3(-3-1)(-3-2)\ldots(-3-r+1)}{r!}(-x^2)^r$ $\qquad (-x^2)^r = \{(-1)x^2\}^r = (-1)^r x^{2r}$

$= \dfrac{-3(-4)(-5)\ldots(-r-2)}{r!}(-1)^r x^{2r}$

$T_{r+1} = (-1)^r \dfrac{3(4)(5)\ldots(r+2)}{r!}(-1)^r x^{2r}$

$$T_{r+1} = (-1)^r \frac{3(4)(5)\cdots(r+2)}{r!}(-1)^r x^{2r}$$

$$T_{r+1} = (-1)^{2r} \frac{(1)(2)\,3(4)(5)\cdots(r+2)}{(1)(2)\,r!} x^{2r}$$

$$T_{r+1} = \frac{(1)(2)\,3(4)(5)\cdots\cdots r(r+1)(r+2)}{(1)(2)\,r!} x^{2r} \quad ; (-1)^{2r} = 1$$

$$T_{r+1} = \frac{r!\,(r+1)(r+2)}{2\,r!} x^{2r} \quad ; 1\times2\times3\times4\times5 = 5!$$

$$T_{r+1} = \frac{(r+1)(r+2)}{2} x^{2r}$$ Ans

Verify your solution

How to verify your solution ?

$$T_{r+1} = \frac{(r+1)(r+2)}{2} x^{2r}$$

For $r = 0$ $\quad T_{0+1} = T_1 = \frac{(0+1)(0+2)}{2} x^{2(0)} = \frac{(1)(2)}{2} 1 = 1$

For $r = 1$ $\quad T_{1+1} = T_2 = \frac{(1+1)(1+2)}{2} x^{2(1)} = \frac{(2)(3)}{2} x^{2(1)} = 3x^2$

For $r = 2$ $\quad T_{2+1} = T_3 = \frac{(2+1)(2+2)}{2} x^{2(2)} = \frac{(3)(4)}{2} = 6x^4$

for $r = 3$ $\quad T_{3+1} = T_4 = \frac{(3+1)(3+2)}{2} x^{2(3)} = \frac{(4)(5)}{2} x^{2(3)} = 10x^6$

$$(1+(-x^2))^{-3} = 1 + (-3)(-x^2) + \frac{(-3)(-4)}{2!}(-x^2)^2 + \frac{(-3)(-4)(-5)}{3!}(-x^2)^3 + ---$$

$$(1+(-x^2))^{-3} = 1 + 3x^2 + 6x^4 + 10x^6 + ---$$

Correction credit: Ashutosh Chauhan (CSE GP MAU)

$$n-(r-1)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots \infty \text{ terms}$$

$$T_1 \quad T_2 \quad T_3 \quad T_4 \quad \dots \quad T_{r+1}$$

$$T_6 = \frac{n(n-1)(n-2)(n-3)(n-4)x^5}{5!}$$

Q.5: $(4-3x)^{-1/2}$ के प्रसार में x^5 का गुणांक बतायें।

Find the coefficient of x^5 in the expansion of (4 - 3x)^(-1/2)

Solution :

$$(4-3x)^{-1/2} = \left\{4\left(1-\frac{3}{4}x\right)\right\}^{-1/2} = 4^{-1/2}\left[1+\left(-\frac{3}{4}x\right)\right]^{-1/2}$$

$$4^{-1/2}\left[1+\left(-\frac{3}{4}x\right)\right]^{-1/2}$$

$$T_6 = 4^{-1/2}\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}-1\right)\left(-\tfrac{1}{2}-2\right)\left(-\tfrac{1}{2}-3\right)\left(-\tfrac{1}{2}-4\right)\left(-\frac{3}{4}x\right)^5 \cdot \frac{1}{5!}$$

$$= \frac{1}{4^{1/2}}\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\left(-\tfrac{5}{2}\right)\left(-\tfrac{7}{2}\right)\left(-\tfrac{9}{2}\right)(-1)\frac{3^5}{4^5}x^5 \frac{1}{5!}$$

$$T_6 = 7\times\frac{3^7}{2^{19}}x^5$$

$$\text{coefficient of } x^5 = 7\times\frac{3^7}{2^{19}}$$

$${}^{n}C_{r} = {}^{n}C_{n-r}$$

$${}^{n}C_{x} = {}^{n}C_{y} \; ; \; x \neq y$$

Then $x + y = n$

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Q.6: ${}^{10}C_{r} = {}^{10}C_{r+2}$ तो ${}^{10}C_{r}$ का मान बतायें

$$r + r + 2 = 10$$

$$2r + 2 = 10$$

$$r = 4$$

$${}^{10}C_4 = \frac{10!}{4!\,(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4 \times 3 \times 2 \times 1 \times 6!}$$

$${}^{10}C_r = {}^{10}C_4 = 210$$ Ans

Q.7: $\dfrac{{}^{15}C_r}{{}^{15}C_{r-1}} = \dfrac{11}{5}$ तो r का मान बताये ।
(then find the value of r)

Solution:

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{11}{5}$$

$$5\ {}^{15}C_r = 11\ {}^{15}C_{r-1}$$

$$5\ \frac{15!}{r!\,(15-r)!} = 11\ \frac{15!}{(r-1)!\,(15-r+1)!}$$

$$\frac{5}{r\,(r-1)!\,(15-r)!} = \frac{11}{(r-1)!\,(15-r+1)\,(15-r)!}$$

$$\frac{5}{r(r-1)!\,(15-r)!} = \frac{11}{(r-1)!\,(15-r+1)\,(15-r)!}$$

$5 \times 4 \times 3 \times 2 \times 1$

$= 5 \times 4!$

$$\frac{5}{r} = \frac{11}{15-r+1}$$

$$75 - 5r + 5 = 11r$$

$$80 = 16r$$

$$5 = r$$

$$r = 5 \quad \text{Ans}$$

Verify your solution: do it by yourself

Summary of Binomial theorem:

Binomial theorem for positive integer index: (धनात्मक पूर्णांक के लिए द्विपद प्रमेय)

$$(x+a)^n = {}^nC_0\, x^{n-0} a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + \cdots + {}^nC_r\, x^{n-r} a^r + \cdots + {}^nC_n\, x^{n-n} a^n$$

$$T_{r+1} = {}^nC_r\, x^{n-r} a^r$$

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \cdots + {}^nC_r\, x^r + \cdots + {}^nC_n\, x^n$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots + x^n$$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

Binomial theorem for negative integer or positive or negative fractional index :

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$|x| < 1$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

Summary of application to approximation:

Binomial theorem for positive integer index: (धनात्मक पूर्णांक के लिए द्विपद प्रमेय)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots + x^n$$

If $x << 1$ then

$x = 0.01 \quad x^2 = 0.01^2 = 0.0001$

$$(1+x)^n \cong 1 + nx$$

Binomial theorem for negative integer or positive or negative fractional index :

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$|x| < 1$

If $x << 1$ then

$$(1+x)^n \cong 1 + nx$$

Q. 8 : $(1.003)^4$ का निकटतम मान ज्ञात करो ।

Find the approximate value of (1.003)^4

Solution :

$$(1.003)^4 = (1 + 0.003)^4$$

$$(1+x)^n \cong 1 + nx \; ; x << 1$$

$$\cong 1 + 4\,(0.003)$$

$$\cong 1 + 0.012$$

$$\cong 1.012$$

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