Topic: Binomial theorem for negative integer or positive or negative fractional index :

PDF link : Description

For any error: Read pin comment

2 Corrections to previous video:

(1) Page number 19 (Question 2) Time 11:24

(2) Page number 26 (Question 6) Time 34:35

Binomial theorem for positive integer index :

$$(x+a)^n = {}^nC_0\, x^{n-0} a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + \dots + {}^nC_r\, x^{n-r} a^r + \dots + {}^nC_n\, x^{n-n} a^n$$

$$(x+a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} a + {}^nC_2\, x^{n-2} a^2 + \dots + {}^nC_r\, x^{n-r} a^r + \dots + {}^nC_n\, a^n$$

$$(a+x)^n = {}^nC_0\, a^{n-0} x^0 + {}^nC_1\, a^{n-1} x^1 + {}^nC_2\, a^{n-2} x^2 + \dots + {}^nC_r\, a^{n-r} x^r + \dots + {}^nC_n\, a^{n-n} x^n$$

$$(a+x)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} x + {}^nC_2\, a^{n-2} x^2 + \dots + {}^nC_r\, a^{n-r} x^r + \dots + {}^nC_n\, x^n$$

If $a = 1$

$$(1+x)^n = {}^nC_0\, 1^n + {}^nC_1\, 1^{n-1} x + {}^nC_2\, 1^{n-2} x^2 + \dots + {}^nC_r\, 1^{n-r} x^r + \dots + {}^nC_n\, x^n$$

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \dots + {}^nC_r\, x^r + \dots + {}^nC_n\, x^n$$

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \dots + {}^nC_r\, x^r + \dots + {}^nC_n\, x^n \quad \text{--- (1)}$$

Binomial coefficients: (द्विपद गुणांक): $^nC_r = \dfrac{n!}{r!\,(n-r)!}$

$0! = 1$

$1! = 1$

$$^nC_0 = \frac{n!}{0!\,n!} = \frac{1}{0!} = 1$$

$$^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n(n-1)!}{1\,(n-1)!} = n$$

$5! = 5\times4\times3\times2\times1$

$= 5\times4!$

$$^nC_2 = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)(n-2)!}{2!\,(n-2)!} = \frac{n(n-1)}{2!}$$

$5! = 5\times4\times3\times2\times1$

$= 5\times4\times3!$

$$^nC_3 = \frac{n!}{3!\,(n-3)!} = \frac{n(n-1)(n-2)(n-3)!}{3!\,(n-3)!} = \frac{n(n-1)(n-2)}{3!}$$

$\vdots$

$$^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)\,(n-r)!}{r!\,(n-r)!}$$

$$^nC_n = \frac{n!}{n!\,(n-n)!}$$

$$^nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

Binomial theorem for positive integer index :

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \cdots + {}^nC_r\, x^r + \cdots + {}^nC_n\, x^n \quad \text{--- (1)}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots + x^n$$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_{r+1} \quad T_{n+1}$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

let $r = 3$

$$T_{3+1} = T_4 = \frac{n(n-1)(n-2)}{3!}x^3$$

$4 - 2 = 2$

$n(n-1)(n-2)$

Binomial theorem for negative integer or positive or negative fractional index :

(ऋणात्मक पूर्णांक या धनात्मक या ऋणात्मक भिन्न के लिए द्विपद प्रमेय)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots \infty \text{ terms}$$

T_1 T_2 T_3 T_4 T_{r+1}

$$|x| < 1 \Rightarrow -1 < x < 1$$

$$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\dots[n-(r-1)]\,x^r}{r!}$$

$$(r+1) - (r-1)$$

$$r+1-r+1 = 2$$

Properties of expansion: (विस्तार के गुण)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$|x| < 1 \Rightarrow -1 < x < 1$

n : negative integer or positive or negative fraction

(ऋणात्मक पूर्णांक या धनात्मक या ऋणात्मक भिन्न)

(1) Number of terms: ∞ terms

(2) ${}^nC_r$: have no meaning when n is negative integer or positive or negative fraction

(3) First term always 1

$(x+a)^n \; ; \; x > a$

$= x^n\left[1 + \frac{a}{x}\right]^n$

(4) General term of given expansion: (विस्तार का व्यापक पद):

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

$$T_4 = \frac{n(n-1)(n-2)}{3!}x^3$$

Binomial theorem for negative integer or positive or negative fractional index :

(ऋणात्मक पूर्णांक या धनात्मक या ऋणात्मक भिन्न के लिए द्विपद प्रमेय)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$$(1-x)^n = \{1+(-x)\}^n$$

$$(1+x)^{-n}$$

$$(1-x)^{-n}$$

Application to approximation: (सन्निकट मान ज्ञात करना): n : Negative Integer or Fraction.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$|x| < 1$

$x = 0.01 \quad x^2 = (0.01)^2 = 0.0001$

If $x^2, x^3, x^4 \ldots$ are very small as compare to x

(यदि $x^2, x^3, x^4 \ldots x$ की तुलना में बहुत कम हैं।)

$$(1+x)^n \cong 1 + nx$$

Example : $(1.01)^{1/2} = (1+0.01)^{\frac{1}{2}} \cong 1 + \frac{1}{2}(0.01)$

$\cong 1 + 0.005$

$\cong 1.005$

From calculator $(1.01)^{1/2} = 1.004987562$

$\cong 1.005$

Binomial coefficients: (द्विपद गुणांक):

$${}^nC_0 = \frac{n!}{0!\,n!} = \frac{1}{0!} = 1$$

$${}^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n(n-1)!}{1\,(n-1)!} = n$$

$${}^nC_2 = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)(n-2)!}{2!\,(n-2)!} = \frac{n(n-1)}{2!}$$

$${}^nC_3 = \frac{n!}{3!\,(n-3)!} = \frac{n(n-1)(n-2)(n-3)!}{3!\,(n-3)!} = \frac{n(n-1)(n-2)}{3!}$$

$$\vdots$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)\,(n-r)!}{r!\,(n-r)!}$$

$${}^nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

$${}^nC_n = \frac{n!}{n!\,(n-n)!} = \frac{1}{0!} = 1$$

$${}^nC_{n-1} = \frac{n!}{(n-1)!\,(n-n+1)!} = \frac{n(n-1)!}{(n-1)!\,1!} = n$$

$${}^nC_{n-2} = \frac{n!}{(n-2)!\,(n-n+2)!} = \frac{n(n-1)(n-2)!}{(n-2)!\,2!} = \frac{n(n-1)}{2!}$$

$$\vdots$$

$${}^nC_r = {}^nC_{n-r}$$

$${}^nC_0 = {}^nC_{n-0} = {}^nC_n$$

$${}^nC_1 = {}^nC_{n-1}$$

$${}^nC_2 = {}^nC_{n-2}$$

Result (1): $^{n}C_{r} = {}^{n}C_{n-r}$

$$\underbrace{r + n - r}_{} = n$$

Result (2): $^{n}C_{x} = {}^{n}C_{y}$; $x \neq y$

Then $x + y = n$

Summary:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \cdots \infty \text{ terms}$$

$T_1 \quad T_2 \quad T_3 \quad T_4$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r = \frac{n(n-1)(n-2)\ldots[n-(r-1)]}{r!}x^r$$

$$T_6 = \frac{n(n-1)(n-2)(n-3)(n-4)x^5}{5!}$$

Coefficient of x^r

$$^nC_r = {^nC_{n-r}}$$

$$^nC_x = {^nC_y} \; ; \; x \neq y$$

Then $x + y = n$

Next video: Problems based on above formulae

COMMENT

LIKE

SHARE

SUBSCRIBE

YouTube Channel

Khushi Foundation Academy

THANK YOU