Topic: Binomial theorem

Problems based on………..

- Find any term in binomial expansion
- Middle term(s) in the expansion of (x + a)^n
- To find the coefficient of x^r in expansion of (x + a)^n
- Term independent of x or constant term in the expansion of (x + a)^n

PDF link : Description

For any error: Read pin comment

Binomial theorem for positive integer index: (धनात्मक पूर्णांक के लिए द्विपद प्रमेय)

$$(x+a)^n = \underbrace{{}^nC_0\, x^{n-0}\, a^0}_{T_1} + \underbrace{{}^nC_1\, x^{n-1}\, a^1}_{T_2} + \underbrace{{}^nC_2\, x^{n-2}\, a^2}_{T_3} + \cdots + \underbrace{{}^nC_r\, x^{n-r}\, a^r}_{T_{r+1}} + \cdots + {}^nC_n\, x^{n-n}\, a^n$$

General term in the expansion of (x + a)^n $(x+a)^n$ के विस्तार में व्यापक पद :

$(r+1)^{th}$ Term $\qquad T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$(r+1)$ वाँ पद

$(x+a)^n$ के विस्तार में $(r+1)$ वाँ पद $\quad T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$\left(\frac{xy}{z}\right)^m = \frac{x^m y^m}{z^m} \quad ; \quad (x^m)^n = x^{mn} \quad ; \quad (-1)^r = \begin{cases} 1 & ; r : \text{even (सम)} \\ -1 & ; r : \text{odd (विषम)} \end{cases}$$

Q.1: $(x+a)^{10}$ के विस्तार में 7 वाँ पद ज्ञात करें।

Find 7th term in the expansion of (x + a)^10

Solution :

$(x+a)^n$ के विस्तार में $(r+1)$ वाँ पद $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$$T_7 = {}^{10}C_6\, x^{10-6}\, a^6$$

$$= \frac{10!}{6!\,(10-6)!}\, x^4 a^6$$

$$= \frac{10 \times \overset{3}{9} \times 8 \times 7 \times 6!}{6!\; 4 \times 3 \times 2 \times 1}\, x^4 a^6$$

$$T_7 = 210\, x^4 a^6$$

Ans

Q.2: $(a+2x^3)^{16}$ के विस्तार में 14 वाँ पद होगा ।

What will be the 14th term in the expansion of (a+2x^3)^16

(a) $560\, a^{13}\, 2^3\, x^9$ (b) $590\, a^3\, 2^{13}\, x^{39}$ (c) $560\, a^3\, 2^{13}\, x^{39}$ (d) कोई नहीं

Solution :

$(x+a)^n$ के विस्तार में $(r+1)$ वाँ पद $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$T_{14} = {}^{16}C_{13}\, (a)^{16-13}\, (2x^3)^{13}$

$= {}^{16}C_{13}\, a^3\, 2^{13}\, (x^3)^{13}$

$T_{14} = {}^{16}C_{13}\, a^3\, 2^{13}\, x^{39}$

$= \dfrac{16!}{13!\,(16-13)!}\, a^3\, 2^{13}\, x^{39}$

$= \dfrac{16 \times \overset{5}{15} \times \overset{7}{14} \times 13!}{13!\; 3 \times 2 \times 1}\, a^3\, 2^{13}\, x^{39}$

$35 \times 16 = 560$

$= 560 a^3\, 2^{13}\, x^{39}$

Correction credit: Vishal Bharati (CSE, GP Mau)

Q. 3 : $\left[\frac{4x}{5} - \frac{5}{4x}\right]^{10}$ के प्रसार में छठा पद ज्ञात करें।

Find 6th term in the expansion of $\left[\frac{4x}{5} - \frac{5}{4x}\right]^{10}$

Solution :

$(x+a)^n$ के विस्तार में $(r+1)$ वाँ पद $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$$\left[\frac{4x}{5} - \frac{5}{4x}\right]^{10} = \left[\frac{4x}{5} + \left(-\frac{5}{4x}\right)\right]^{10}$$

$$T_6 = {}^{10}C_5 \left(\frac{4x}{5}\right)^{10-5} \left(\frac{-5}{4x}\right)^5$$

$$[-5]^5 = [(-1)\,5]^5 = (-1)^5\, 5^5$$

$$= -\frac{10!}{5!\,(10-5)!}\,\frac{4^5 x^5}{5^5}\,\frac{5^5}{4^5 x^5} = -\frac{10\times 9\times 8\times 7\times 6\times 5!}{5!\ \ 5\times 4\times 3\times 2\times 1}$$

$$T_6 = -252$$

Middle term(s) in the expansion of (x + a)^n

$(x+a)^n$ के विस्तार में मध्य पद)

Case 1:when n is even:

(यदि n सम हो)

$$\left[\frac{n+2}{2}\right]^{th} \text{ term} = \left[\frac{n}{2}+1\right] \text{ term}$$

Case 1:when n is odd:

(यदि n विषम हो)

$$\left[\frac{n+1}{2}\right]^{th} \text{ term and } \left[\frac{n+3}{2}\right]^{th} \text{ term}$$

Q.4: $\left[\frac{4x}{5a}+\frac{5a}{4x}\right]^{10}$ के विस्तार का मध्य पद प्राप्त करें।

Find the middle term(s) in the expansion of $\left[\frac{4x}{5a}+\frac{5a}{4x}\right]^{10}$

Solution : $n = 10$ even | middle term

$\left[\frac{n+2}{2}\right]^{th}$ term $= \left[\frac{10+2}{2}\right]^{th}$ term $=$ 6^{th} term

$$T_6 = {}^{10}C_5 \left(\frac{4x}{5a}\right)^{10-5} \left(\frac{5a}{4x}\right)^5$$

$$= \frac{10!}{5!(10-5)!} \frac{4^5 x^5}{5^5 a^5} \cdot \frac{5^5 a^5}{4^5 x^5}$$

$$= \frac{10^2 \times 9 \times 8 \times 7 \times 6^2 \times 5!}{5! \; 5 \times 4 \times 3 \times 2 \times 1} = 252 \Rightarrow \boxed{T_6 = 252}$$

Q.5: $\left[\frac{4x}{5a}-\frac{5a}{4x}\right]^{12}$ के विस्तार का मध्य पद प्राप्त करें।

Find the middle term(s) in the expansion of $\left[\frac{4x}{5a}-\frac{5a}{4x}\right]^{12}$

Solution : $n = 12$ even (सम), 1 Middle term (मध्य पद)

$$\left[\frac{n+2}{2}\right]^{th} \text{term} = \left[\frac{12+2}{2}\right]^{th} \text{term} = 7^{th} \text{ term}$$

T_7

$(x+a)^n$ के विस्तार में $(r+1)$ वॉ पद $T_{r+1} = {}^nC_r \, x^{n-r} \, a^r$

$$\left[\frac{4x}{5a}-\frac{5a}{4x}\right]^{12} = \left[\frac{4x}{5a}+\left(-\frac{5a}{4x}\right)\right]^{12}$$

$$T_7 = {}^{12}C_6 \left(\frac{4x}{5a}\right)^{12-6} \left(\frac{-5a}{4x}\right)^6$$

$$T_7 = {}^{12}C_6 \left(\frac{4x}{5a}\right)^{12-6} \left(\frac{-5a}{4x}\right)^6$$

$$= \frac{12!}{6!(12-6)!} \frac{4^6 x^6}{5^6 a^6} \frac{5^6 a^6}{4^6 x^6}$$

$; (-1)^6 = 1$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!\ 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$T_7 = 924$$

Q.6: $\left(2x - \dfrac{x^2}{4}\right)^9$ के प्रसार में मध्य पद ज्ञात करें।

Find the middle term(s) in the expansion of $\left(2x - \dfrac{x^2}{4}\right)^9$

Solution :

$n = 9$ (odd) (विषम) , two middle term (दो मध्य पद)

$\left[\dfrac{n+1}{2}\right]^{th}$ term and $\left[\dfrac{n+3}{2}\right]^{th}$ term

$\left[\dfrac{9+1}{2}\right]^{th}$ term and $\left[\dfrac{9+3}{2}\right]^{th}$ term

5^{th} term and 6^{th} term

T_5 T_6

T_1 T_2 T_3 T_4 T_5 T_6 T_7 T_8 T_9 T_{10}

$$\left(2x - \frac{x^2}{4}\right)^9 = \left\{2x + \left(-\frac{x^2}{4}\right)\right\}^9$$

$$T_5 = {}^9C_4 \, (2x)^{9-4} \left(-\frac{x^2}{4}\right)^4$$

$$= \frac{9!}{4!\,(9-4)!} \, 2^5 x^5 \, \frac{(x^2)^4}{4^4} = \frac{9\times 8\times 7\times 6\times 5!}{(4\times 3\times 2\times 1)\times 5!} \, \frac{2^5 \, x^5 \, x^8}{(2^2)^4}$$

$$T_5 = \frac{63}{4} x^{13}$$

$$T_6 = {}^9C_5 \, (2x)^{9-5} \left(-\frac{x^2}{4}\right)^5$$

$$= \frac{9!}{5!\,(9-5)!} \, 2^4 x^4 \left[-\frac{x^{10}}{4^5}\right] = -\frac{9\times 8\times 7\times 6\times 5!}{5!\; 4\times 3\times 2\times 1} \, 2^4 \, x^4 \, \frac{x^{10}}{(2^2)^5}$$

$$T_6 = -\frac{63}{32} x^{14}$$

Correction credit: Raunak Kumar (Electronics Engineering , GP Mau)

Q.7: $\left(x^2+\frac{1}{x^5}\right)^{12}$ के प्रसार में x^3 का गुणांक ज्ञात करें।

Find the coefficient of x^3 in the expansion of $\left(x^2+\frac{1}{x^5}\right)^{12}$

Solution : $(p+1)^{th}$ term have x^3

$$T_{p+1} = {}^{12}C_p \left(x^2\right)^{12-p} \left(\frac{1}{x^5}\right)^p$$

$$= {}^{12}C_p \; x^{24-2p} \frac{(1)^p}{x^{5p}}$$

$$T_{p+1} = {}^{12}C_p \; x^{24-7p} \quad \text{—} (1)$$

$$24-7p = 3$$

$$21 = 7p$$

$$3 = p \Rightarrow p=3 \Rightarrow (p+1)^{th} \text{ term} = 3+1 = 4^{th} \text{ term}$$

$$T_{p+1} = {}^{12}C_p \, x^{24-7p}$$

$$T_{3+1} = T_4 = {}^{12}C_3 \, x^{24-7(3)}$$

$$T_4 = {}^{12}C_3 \, x^3$$

x^3 Coefficient $= {}^{12}C_3$

$$= \frac{12!}{3!\,(12-3)!} = \frac{\overset{2}{12} \times 11 \times 10 \times 9!}{3 \times 2 \times 1 \times 9!}$$

$$= 220$$

Q.8: $\left(\frac{4x}{5} + \frac{5}{2x}\right)^{17}$ के प्रसार में x^5 का गुणांक ज्ञात करें।

Find the coefficient of x^5 in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x}\right)^{17}$

Solution : let $(p+1)^{th}$ term have x^5

$$T_{p+1} = {}^{17}C_p \left(\frac{4x}{5}\right)^{17-p} \left(\frac{5}{2x}\right)^p$$

$$= {}^{17}C_p \frac{4^{17-p}\, x^{17-p}}{5^{17-p}} \frac{5^p}{2^p\, x^p}$$

$$T_{p+1} = {}^{17}C_p \frac{4^{17-p}\, 5^p}{5^{17-p}\, 2^p} x^{17-2p} \quad \text{——} \; (1)$$

Given $17 - 2p = 5$

$$17-2p=5$$
$$17-5=2p$$
$$12=2p$$
$$6=p$$
$$p=6$$

$(p+1)^{th}$ term $=(6+1)^{th}$ term $=7^{th}$ term

From eq$^{(n)}$ (1) we have

$$T_{p+1}={}^{17}C_p\,\frac{4^{17-p}\ 5^{p}}{5^{17-p}\ 2^{p}}\,x^{17-2p}$$

$$T_{6+1}=T_7={}^{17}C_6\,\frac{4^{17-6}}{5^{17-6}}\,\frac{5^6}{2^6}\,x^{17-2(6)}={}^{17}C_6\,\frac{4^{11}}{5^5}\cdot\frac{1}{2^6}\,x^5$$

$$2^{22}$$

$$T_{6+1}={}^{17}C_6\,\frac{2^{16}}{5^5}\,x^5 \quad ; \quad 4^{11}=(2^2)^{11}=2^{22}$$

Coefficient of x^5 $={}^{17}C_6\,\frac{2^{16}}{5^5}$ Ans

Q.9: $\left(x-\frac{1}{x}\right)^{12}$ के विस्तार में x से मुक्त पद (अचर पद) ज्ञात करें।

Find the term independent of x (constant term) in the expansion of $\left(x-\frac{1}{x}\right)^{12}$

Solution : let $(p+1)^{th}$ term

$$T_{p+1} = {}^{12}C_p (x)^{12-p} \left(-\frac{1}{x}\right)^p$$

$$= {}^{12}C_p \; x^{12-p} \; \frac{(-1)^p}{x^p}$$

$$T_{p+1} = (-1)^p \; {}^{12}C_p \; x^{12-2p} \quad \text{—} \; (1) \qquad x^0$$

Given : $12-2p = 0$

$$-2p = -12$$

$$p = 6$$

$(p+1)^{th}$ term $= (6+1)^{th}$ term $= 7^{th}$ term

$$T_{p+1} = (-1)^p \; {}^{12}C_p \; x^{12-2p}$$

$$T_{6+1} = T_7 = (-1)^6 \; {}^{12}C_6 \; x^{12-2(6)}$$

$$T_7 = {}^{12}C_6 \; x^0$$

$$= \frac{12!}{6!\,(12-6)!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6! \; 6 \times 5 \times 3 \times 2 \times 1}$$

$$T_7 = 924$$

Q.10: $\left(2x^4-\frac{1}{3x^7}\right)^{11}$ के विस्तार में x से मुक्त पद (अचर पद) ज्ञात करें।

Find the term independent of x (constant term) in the expansion of $\left(2x^4-\frac{1}{3x^7}\right)^{11}$

Solution : let $(p+1)^{th}$ term is independent of x

$$T_{p+1} = {}^{11}C_p (2x^4)^{11-p} \left(\frac{-1}{3x^7}\right)^p = {}^{11}C_p\, 2^{11-p} (x^4)^{11-p} \frac{(-1)^p}{3^p (x^7)^p}$$

$$= (-1)^p\, {}^{11}C_p \frac{2^{11-p}}{3^p} \frac{x^{44-4p}}{x^{7p}}$$

$$T_{p+1} = (-1)^p\, {}^{11}C_p \frac{2^{11-p}}{3^p} x^{44-11p} \quad \text{——— (1)}$$

Given: $44-11p=0 \Rightarrow 44 = 11p \Rightarrow 4 = p \Rightarrow p = 4$ $\therefore (p+1)^{th}$ term $= 5^{th}$ term

From eq. ① we have

$$T_{p+1} = (-1)^p \, {}^{11}C_p \, \frac{2^{11-p}}{3^p} \, x^{44-11p}$$

$$T_{4+1} = (-1)^4 \, {}^{11}C_4 \, \frac{2^{11-4}}{3^4} \, x^{44-11(4)}$$

$$T_5 = {}^{11}C_4 \, \frac{2^7}{3^4} \, x^0$$

$$T_5 = {}^{11}C_4 \, \frac{2^7}{3^4} = \frac{11!}{4!(11-4)!} \, \frac{128}{81} = \frac{11 \times 10 \times 9 \times 8 \times 7!}{4 \times 3 \times 2 \times 1 \times 7!} \times \frac{128}{81}$$

$$T_5 = \frac{14080}{27}$$ Ans

Next video: Binomial theorem for negative integer and fractional index

COMMENT

LIKE

SHARE

SUBSCRIBE

YouTube Channel

Khushi Foundation Academy

THANK YOU